

# **Risks and Portfolio Decisions involving Hedge Funds**

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**Current Version: July 17, 2002**

**JEL Classification: G10, G19**

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# **Risks and Portfolio Decisions involving Hedge Funds**

## **Abstract**

Hedge funds are known to exhibit non-linear option-like exposures to standard asset classes and therefore the traditional linear factor model provides limited help in capturing their risk-return tradeoffs. We address this problem by augmenting the traditional model with option-based risk factors. Our results show that a large number of equity-oriented hedge fund strategies exhibit payoffs resembling a short position in a put option on the market index, and therefore bear significant left-tail risk, risk that is ignored by the commonly used mean-variance framework. Using a mean-conditional Value-at-Risk framework, we demonstrate the extent to which the mean-variance framework underestimates the tail risk. Working with the underlying systematic risk factors, we compare the long-run performance with the recent performance of hedge funds and find that their recent performance appears significantly better than their long-run performance. Our analysis provides important insights that can be helpful in addressing issues like construction of fund of funds, risk management, benchmark design and manager compensation involving hedge funds.

## Risks and Portfolio Decisions involving Hedge Funds

It is well accepted that the world of financial securities is a multi-factor world consisting of different risk-factors, each associated with its own factor-risk-premium, and that no single investment strategy can span the entire “risk-factor space”. Therefore, investors wishing to earn risk premia associated with different risk-factors need to employ different kinds of investment strategies. Sophisticated investors, like endowments and pension funds, seem to have recognized this fact as their portfolios consist of mutual funds as well as hedge funds<sup>1</sup>. Mutual funds typically employ long only buy-and-hold type strategy on standard asset classes, and help capture risk-premia associated with equity-risk, interest-rate risk, default-risk etc. However, they are not very helpful in capturing risk-premia associated with dynamic trading strategies or spread-based strategies. This is where hedge funds come into the picture. Unlike mutual funds, hedge funds are not evaluated against a passive benchmark and therefore can follow more dynamic trading strategies. Moreover, they can take long as well as short positions in securities, and therefore can bet on Capitalization spreads or Value-Growth spreads. As a result, hedge funds can offer exposure to risk-factors that traditional long-only strategies cannot<sup>2</sup>.

As there is no “free-lunch” in financial markets, question arises regarding the kinds and nature of risks associated with different hedge fund strategies. This is a challenging task given the complex nature of the strategies and limited disclosure requirements faced by hedge funds. Out of

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<sup>1</sup> For example, consider the investment strategies of large endowments like Harvard and Yale, or large pension funds like CALPERS and Ontario Teachers. We know from Fung and Hsieh (1997, 2001a) that mutual funds predominantly employ relatively static trading strategies while hedge funds and CTAs employ relatively dynamic trading strategies. Although they trade in similar asset classes as mutual funds, they show relatively low correlation with long-only type strategies.

<sup>2</sup> Although, in principle, investors can create exposure like hedge funds by trading on their own account, in practice they encounter many frictions due to incompleteness of markets like the publicly traded derivatives market and the financing market. Although derivatives market for standardized contracts has grown a great deal in recent years, it is still very costly for an investor to create a customized payoff on individual securities. The same is true of the financing market as well where investors encounter difficulties shorting securities and obtaining leverage. These frictions make it difficult for investors to create hedge-fund-like payoffs by trading on their own accounts.

a wide range of hedge fund strategies available in the marketplace, our knowledge to-date is limited to the risks of two strategies: “trend-following” analyzed by Fung and Hsieh (2001a) and “risk-arbitrage” studied by and Mitchell and Pulvino (2001). Both studies find the risk-return characteristics of the hedge fund strategies to be nonlinear, and stress the importance of taking into account option-like features inherent while analyzing hedge funds.

We start with these insights and contribute to this emerging literature in several important ways. First, we extend our understanding of hedge fund risks to a wide range of equity-oriented hedge fund strategies. Instead of imposing a specific functional form, we allow for a flexible piecewise linear function of the market return to approximate the nonlinear payoffs of different hedge fund strategies. Our approach has the advantage that it is an operationally convenient method that can empirically characterize the risk of any generic hedge fund strategy. Second, we examine the implications of nonlinear option-like payoffs of hedge funds for portfolio decisions. We show how the Conditional Value-at-Risk (CVaR) framework, which explicitly accounts for the negative tail risk, can be applied to construct portfolios involving hedge funds<sup>3</sup>. We contrast our results with those obtained using the traditional mean-variance framework. Finally, we show how the limitation of short history of hedge fund returns can be overcome by working with the underlying risk factors estimated through a multi-factor model<sup>4</sup>. Since the underlying risk factors have longer return history, this approach can provide insights into the long-term risk-return tradeoffs of hedge funds. On the whole, it provides important insights into the different hedge fund strategies, insights that are very helpful while taking investment decisions like portfolio construction, risk management, benchmark design, manager compensation etc. involving hedge funds.

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<sup>3</sup> CVaR corresponds to the statistical mean of losses exceeding the VaR. While the VaR focuses only on the frequency of extreme events, CVaR focuses on both frequency and size of losses in case of extreme events.

<sup>4</sup> This is in the spirit of asset-based style factors proposed by Fung and Hsieh (2001b).

It is well known that payoffs of managed portfolio will show option-like features (see Merton (1981) and Dybvig and Ross (1985)). The importance of taking into account such option-like features, even when the fund manager does not have superior information and does not trade in derivatives, was first demonstrated by Jagannathan and Korajczyk (1986). The focus of this earlier stream of research was on assigning a value to the superior information that a skilled portfolio manager may possess by separating the skill into two dichotomous categories: market timing and security selection. Glosten and Jagannathan (1994) were the first to point out that even though it is rather difficult to separate a manager's ability clearly into two such categories, it is still possible to characterize the nature of the risk in managed portfolios and assign an overall value to the manager's skills by using derivative pricing methods. They suggested the inclusion of "...excess returns on certain selected options on stock index portfolios as additional 'factor excess returns'." Our paper builds on this established theoretical framework supported by recent empirical evidence of option-like features in hedge fund payoffs<sup>5</sup>. Our use of exchange-traded options offers several advantages. First, they help capture the hedge fund risks in an intuitive manner. Second, being based on market prices, they embed investor preferences, information and market conditions. Finally, being highly liquid and exchange-traded, they enable replication of hedge fund payoffs.

We propose a two-step approach to characterize hedge fund risks. In the first step, we estimate the risk exposures of hedge funds using a multi-factor model consisting of excess returns on standard assets and options on these assets as risk factors. In the second step, we examine the ability of these risk factors to replicate the out-of-sample performance of hedge funds. Our out-of-sample analysis confirms that the risk factors estimated in the first step are not statistical artifacts

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<sup>5</sup> Hedge funds provide an ideal testing ground for the application of Glosten and Jagannathan's (1994) approach due to several reasons, some of which do not arise in case of mutual funds analyzed by them. This is because, unlike most mutual funds (see Koski and Pontiff (1999) and Almazan et al (2001)), hedge funds frequently trade in derivatives. Second, hedge funds are known for their 'opportunistic' nature of trading

of the data, but represent underlying economic risk exposures of hedge funds. Application of our approach at the hedge fund index level captures the “popular bets” taken (i.e., common risks borne) by a large number of hedge funds that were operating during the sample period, while application at the individual hedge fund level provides information about the systematic risks borne by that specific hedge fund.

Hedge funds may exhibit non-normal payoffs for various reasons such as their use of options, or option-like dynamic trading strategies or strategies that lose money during market downturns. For example, during the Russian debt crisis in August 1998 a wide range of hedge funds reported large losses. This suggests that hedge funds may be bearing significant left-tail risk. Regulatory bodies such as the Basle committee have recognized this feature and have emphasized the importance of tail risk and use of risk management frameworks such as the Value-at-Risk (VaR). Keeping this in mind, we employ a mean-conditional value at risk (M-CVaR) framework for portfolio construction involving hedge funds. Using this framework, we examine the extent to which traditional mean-variance framework underestimates the tail risk of hedge funds.

We address the common problem of short history of hedge fund returns one encounters while conducting empirical research on hedge funds. Since most hedge fund databases report their returns from early nineties, a natural question arises as to how the hedge funds would have performed during extreme events in the past, such as the Great Depression of the 1930’s, the oil shock of the early 1970’s, or the stock market crash of 1987. We shed light on this issue by working with the underlying risk factors that have longer return history. Assuming that the hedge funds were bearing the same systematic risk exposures as those during the nineties, we estimate their returns prior to our sample period and compare their long-term performance with their performance during the nineties. We show how this approach can help investors get a long-term perspective on the risk-return tradeoffs of hedge funds.

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and a significant part of their returns arise from taking state-contingent bets.

Our analysis provides three main findings. First, we find that the non-linear option-like payoffs are not restricted only to “trend-followers” and “risk-arbitrageurs”, but are an integral feature of the payoffs on a wide range of hedge fund strategies. In particular, we observe that the payoffs on a large number of equity-oriented hedge fund strategies resemble those from writing a put option on the equity index. Second, we find that the expected tail losses of mean-variance optimal portfolios can be underestimated by as high as 54% compared to mean-CVaR optimal portfolios. This suggests that ignoring the tail risk of hedge funds can result in significantly higher losses during large market downturns. Finally, our analysis using extrapolated hedge fund returns during 1927-1989 period suggests that their performance during the last decade is not representative of their long-term performance. In particular, we find that the expected losses beyond VaR during the 1927-1989 period can be about twice of those during the nineties. We also find that their mean returns during the 1927-1989 period are significantly lower and their standard deviations are significantly higher compared to those of their recent performance. These findings have important implications for risk management and portfolio decisions involving hedge funds. They also provide support to the theoretical modeling of hedge funds in Kyle and Xiong (2001) framework.

Rest of the paper is organized as follows. Section 1 provides the theoretical framework. Section 2 contains the description of data and the risk factors (buy-and-hold and option-based) used in our multi-factor model. Section 3 presents the model, the in-sample analysis and various robustness checks while Section 4 conducts the out-of-sample analysis. Section 5 develops the Mean-Conditional VaR framework and contrasts the findings with the traditional mean-variance framework. Section 6 examines the long-term performance of hedge funds and compares it with their recent performance. Section 7 offers concluding remarks and suggestions for future research.

## 1. Theoretical Framework

Linear factor models such as the CAPM and the APT have been the foundation of most of the theoretical and empirical asset pricing literature. Dybvig and Ingersoll (1982) argue that since these theories constrain the relation between risk factors and returns be linear, they cannot price securities whose payoffs are non-linear functions of the risk factors. Bansal and Viswanathan (1993) and Bansal, Hsieh and Viswanathan (1993) circumvent this problem by proposing a non-linear APT framework. They start from the optimal portfolio allocation conditions of discrete time capital asset pricing models, namely,

$$E\left[m_{t,t+1}x_{i(t,t+1)} \mid \mathbb{F}_t\right] = \Pi\left(x_{i(t,t+1)}\right) \quad (1)$$

where  $x_{i(t,t+1)}$  is the payoff of the  $i^{\text{th}}$  asset at time  $t+1$  that has a price  $\Pi\left(x_{i(t,t+1)}\right)$  at time  $t$ ,  $m_{t,t+1}$  is the marginal rate of substitution of the investor from time  $t$  to  $t+1$ , and  $\mathbb{F}_t$  is the information set of investor at time  $t$ . The projection  $p_{t+1}^*$  of the one-period marginal rate of substitution  $m_{t,t+1}$  on the space of the one-period payoffs also satisfies a condition that<sup>6</sup>

$$E\left[p_{t+1}^*x_{i(t,t+1)} \mid \mathbb{F}_t\right] = \Pi\left(x_{i(t,t+1)}\right). \quad (2)$$

In order to address the dimensionality problem, they impose a sufficient statistic restriction on the conditional expectation of  $m_{t,t+1}$ . They use the law of iterated expectations and rewrite equation (1) as

$$E\left[E\left[m_{t,t+1} \mid \mathbb{F}_{t+1}\right]x_{i(t,t+1)} \mid \mathbb{F}_t\right] = \Pi\left(x_{i(t,t+1)}\right). \quad (3)$$

Then, instead of using the projection of  $m_{t,t+1}$  on to the entire information set, they consider a  $K$ -dimensional vector of well-diversified basis variables  $p_{t+1}^b$  such that

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<sup>6</sup> Hansen and Jagannathan (1991) show that this projection  $p_{t+1}^*$  has the minimum variance in the class of all



$$E[m_{t,t+1} | \mathbb{F}_{t+1}] = E[m_{t,t+1} | p_{t+1}^b] = G(p_{t+1}^b), \quad (4)$$

where,  $K$  is a bw number and  $G(\cdot)$  is a well behaved function. As the exact specification of nonlinear pricing kernel is unknown, they approximate it with a polynomial involving second and fifth power of market return<sup>7</sup>

$$G(p_{t+1}^b) = \beta_0 + \beta_{1r} r_{ft+1} + \beta_{1M} r_{Mt+1} + \beta_{2M} r_{Mt+1}^2 + \beta_{5M} r_{Mt+1}^5. \quad (5)$$

Harvey and Siddique (2000a,b) also specify higher powers of market return in the pricing kernel in their investigation of the importance of conditional skewness in asset pricing. In particular, they specify the marginal rate of substitution to be a quadratic function of market return

$$m_{t+1} = a_t + b_t R_{M,t+1} + c_t R_{M,t+1}^2, \quad (6)$$

and derive an asset pricing model of the following form<sup>8</sup>

$$E_t(r_{i,t+1}) = A_t E_t(r_{M,t+1}) + B_t E_t(r_{M,t+1}^2). \quad (7)$$

These studies of asset pricing aim to find the best functional form or specification of the pricing kernel. Their focus is on pricing securities with asymmetric nonlinear payoffs. There exists another strand of literature, which is closely related to the nonlinear payoffs, but which focuses on the use of options to capture and to assign a value to the nonlinearities. In particular, Breeden and Litzenberger (1978) show that any nonlinear payoff can be represented as a combination of options with differing strike prices, while Glosten and Jagannathan (1994) show how a value can be assigned to the skill of the manager generating a nonlinear payoff. In particular, they show that for valuation purposes it is not necessary to replicate the nonlinear payoff by a collection of options, but it is only necessary to replicate that part of the payoff that has nonzero value. For this

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pricing kernels and can be expressed as a linear combination of the one-period payoffs on  $N$  assets.

<sup>7</sup> Bansal and Viswanathan (1993) attribute the use of fifth power instead of the third power to the need to reduce collinearity among different powers of the expansion.

<sup>8</sup> Harvey and Siddique (2000a,b) show that the variation in conditional skewness and coskewness measures is able to explain a significant proportion of the time-series and cross-sectional variation in the equity returns. They also provide explicit expression for the skewness risk premium. See Harvey and Siddique

purpose, it is only necessary to approximate the nonlinear payoff by a collection of options on a selected number of benchmark index returns. There will be some residual risk but that residual risk will not be priced. Glosten and Jagannathan (1994) use the contingent-claim based specification of the form

$$R_p = \alpha + \beta_1 R_m + \beta_2 \max(R_m - k_1, 0) + \beta_3 \max(R_m - k_2, 0) + \beta_4 \max(R_m - k_3, 0) + \varepsilon \quad (8)$$

We build on this framework and specify a flexible piecewise linear form involving call and put options on the market index, namely<sup>9</sup>,

$$R_p = \alpha + \beta_1 R_m + \beta_2 \max(R_m - k_1, 0) + \beta_3 \max(R_m - k_2, 0) + \beta_4 \max(k_1 - R_m, 0) + \beta_5 \max(k_3 - R_m, 0) + \varepsilon \quad (9)$$

Since the returns on options can be expressed as a polynomial function of the market return, our option-based specification is related to the earlier stream of literature expressing the pricing kernel as a polynomial function of market return described in equations (5) and (7) above. In terms of implementation of approach, our augmentation of the linear beta model with nonlinear option-based factors (which have skewed payoffs) is similar in spirit to Harvey and Siddique's (2000b) augmentation of Fama-French's (1993) three-factor model by a nonlinear factor derived from skewness.

Having described the theoretical framework and how our model relates to other nonlinear models used in the literature to price securities with nonlinear payoffs, and to assign a value to the skill of manager generating a nonlinear payoff, we proceed to the description of data and risk factors used in our multi-factor model.

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(1999) for a discussion of the importance of allowing for time-varying conditional skewness.

<sup>9</sup> Our specification encompasses the nonlinear model used by Henriksson and Merton (1981) in the context of market timing ability of managers.

## 2. Description of Data and Risk Factors

In this paper, we analyze equity-oriented hedge fund strategies. The reason for focusing on these strategies is the availability of high quality data on exchange-traded options on broad-based equity indexes such as Standard and Poors' (S&P) 500 Composite index. We analyze six hedge fund strategies whose payoff arises primarily from relative mispricings of securities rather than the movement of the market as a whole, namely, Event Arbitrage, Restructuring, Event Driven, Relative Value Arbitrage, Convertible Arbitrage and Equity Hedge (Long/Short Equity). We also investigate two hedge fund strategies whose payoff arises primarily from taking directional bets, namely, Equity Non-Hedge, and Short Selling (Dedicated Short-Bias). It is well known that hedge fund indexes differ from each other in the way they are constructed<sup>10</sup>. Therefore, we conduct our analysis using both Hedge Fund Research (HFR) and CSFB/Tremont indexes. From the HFR indexes, we select Event Arbitrage, Restructuring, Event Driven, Relative Value Arbitrage, Convertible Arbitrage, Equity Hedge, Equity Non-Hedge and Short Selling indexes. We also select four CSFB/Tremont indexes, namely Event Driven, Convertible Arbitrage, Long/Short Equity and Dedicated Short-Bias that correspond to Event Driven, Convertible Arbitrage, Equity Hedge and Short Selling HFR indexes. Our sample consists of monthly returns on the HFR indexes from January 1990 to June 2000 and on the CSFB/Tremont indexes from January 1994 to June 2000. We validate our findings of economic risk exposures of hedge funds using out-of-sample data from July 2000 to December 2001<sup>11</sup>.

Our multi-factor model uses a set of buy-and-hold and option-based risk factors. The buy-and-hold risk factors consist of indexes representing equities (Russell 3000 index, lagged Russell

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<sup>10</sup> The HFR indexes are equally-weighted and therefore give relatively more weight to the performance of smaller hedge funds while the CSFB/Tremont indexes are value-weighted (i.e. weighted by assets under management) and hence give relatively more weight to the performance of larger hedge funds. See [www.hfr.com](http://www.hfr.com) and [www.hedgeindex.com](http://www.hedgeindex.com) for the index construction details.

<sup>11</sup> We thank the referee for suggesting this approach.

3000 index<sup>12</sup>, MSCI World excluding USA index and MSCI Emerging Markets index), bonds (Salomon Brothers Government and Corporate Bond index, Salomon Brothers World Government Bond index and Lehman High Yield index), Federal Reserve Bank Competitiveness-Weighted Dollar index and the Goldman Sachs Commodity index<sup>13</sup>. We also include three zero-investment strategies representing Fama-French's (1993) "Size" factor (Small-minus-Big or SMB), "Book-to-Market" factor (High-minus-Low or HML) and Carhart's (1997) "Momentum" factor (Winners-minus-Losers)<sup>14</sup>. Finally, to capture credit risk, we include the change in the default-spread (the difference between the yield on the BAA-rated corporate bonds and the ten-year Treasury bonds) as an additional factor.

Our option-based risk factors consist of highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 Composite index trading on the Chicago Mercantile Exchange<sup>15</sup>. Our use of options with different degrees of moneyness allows a flexible piecewise linear risk-return relation<sup>16</sup>. The process of buying an ATM call option on the S&P 500 index works as follows. On the first trading day in January, buy an ATM call option on the S&P 500 index that expires in February. On the first trading day in February, sell the option bought a month ago (i.e. at the beginning of January) and buy another ATM call option on the S&P 500 index that expires in March. Repeating this trading pattern every month provides the

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<sup>12</sup> The use of lagged Russell 3000 index accounts for the effect of non-synchronous trading and is suggested by Asness, Krail and Liew (2001).

<sup>13</sup> One may argue for including specific commodity prices like gold, silver etc. or specific exchange rates like US\$/Yen, US\$/DM, etc. as buy-and-hold factors. We believe that such specific commodity and currency prices may be more appropriate while analyzing risk exposures of Macro funds. However, since our focus is on equity-oriented hedge fund strategies, we consider broad-based commodity and currency indexes.

<sup>14</sup> Edwards and Liew (1999) use a linear factor model consisting of market, size, book-to-market and momentum factors to evaluate the performance of hedge funds. This buy-and-hold model is more general than the traditional linear factor model used earlier in the hedge fund literature and is consistent with the evidence of the four-factor model (the market index, Fama-French's size and book-to-market factors, and Carhart's momentum factor) explaining the returns on stocks and managed portfolios.

<sup>15</sup> If one were to use this approach to analyze the systematic risk of fixed-income-based hedge fund strategies, then one would use bond indexes, spreads between different fixed-income instruments and treasuries, and interest-rate options, options on fixed income securities etc. Similarly, while analyzing Macro funds that take bets on currencies (e.g. Macro), one would include returns on currencies and specific currency options.

time-series of returns on buying an ATM call option. A similar procedure provides time-series of returns on buying OTM call options<sup>17</sup>. We select the ATM option as the one whose present value of strike price is closest to the current index value. We select the OTM call (put) option to be the one with next higher (lower) strike price<sup>18</sup>. We denote ATM call (put) option on the S&P 500 Index by  $SPC_a$  ( $SPP_a$ ) and OTM call (put) option by  $SPC_o$  ( $SPP_o$ ). Using price data from The Institute for Financial Markets, we compute monthly returns to these option-based risk factors.

Our approach has the flexibility to combine long and/or short positions in calls and/or puts with differing strike prices without having to pre-specify whether it is a long or a short position, the number of units of each option, and the strike price of each option<sup>19</sup>. It is this flexibility that enables our option-based risk factors to effectively capture the non-linear payoffs of hedge funds.

We report the summary statistics for the HFR indexes and our buy-and-hold and option-based risk factors during January 1990 to June 2000 period in Panels A and B of Table 1. We also provide the summary statistics of the CSFB/Tremont indexes during January 1994 to June 2000 period in Panel C of Table 1. We show the correlations between the different hedge fund indexes and the risk factors in Table 2. As can be seen, all HFR indexes and three out of four CSFB/Tremont indexes show significant correlation with the Russell 3000 index. A large number of hedge fund indexes also show significant correlation with Fama-French's Size factor. Mitchell

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<sup>16</sup> See Glostien and Jagannathan (1994) for the rationale of using multiple splines with more than one knot.

<sup>17</sup> We do not consider in-the-money (ITM) options as their payoffs can be replicated by a combination of underlying asset and risk-free asset along with an OTM option. For example, the maturity payoff on an ITM call option can be replicated by a long position in the underlying asset, a long position in the risk-free asset and a long position in an OTM put with the same strike price.

<sup>18</sup> Options are available in strike-price increments of five index points. On average, the ratio of index price to present value of strike price for our at-the-money options is 1.00 while that for our out-of-the-money call (put) options is 0.99 (1.01). We discuss the robustness of our results to specifying higher degrees of out-of-the-moneyness in Section 3.2.

<sup>19</sup> We illustrate this with two examples. We know from Fung and Hsieh (1999a) that the returns on Global/Macro funds display a collar like payoff vis-à-vis the US equity index while those on trend-following CTAs exhibit a straddle like payoff on the assets they trade in. So, if one were to regress the Global/Macro fund returns on US equity index and options on the US equity index, then we believe that their collar-like payoff will be manifested through a long position in the index, combined with having written  $x$  units of out-of-the-money calls and having bought  $y$  units of out-of-the money puts on the index. Similarly, if one were to regress the returns of trend-following CTAs on calls and puts on the asset classes they trade in, then we

and Pulvino (2001) find that the risk arbitrage strategy shows zero correlation with the market during up-market conditions but large positive correlation during down-market conditions. In order to examine whether this is true for a wide range of hedge fund indexes, we use a regression specification that allows for separate intercept and slope coefficients when the market index is above and below its median return. We report our findings in Table 3. We find that a large number of hedge fund indexes show no correlation in up-market conditions, but a positive correlation in down-market conditions. This asymmetry of betas or factor loadings in up-market versus down-market conditions confirms the nonlinear nature of hedge fund payoffs. It also suggests that the extent of diversification benefits offered by hedge funds would be smaller during down-market conditions.

### 3. Multi-factor Model and Results

As discussed in the introduction, we employ a two-step procedure to characterize the systematic risk exposures of hedge funds. The first step involves identifying statistically significant factors that ex-post capture in-sample variation in hedge fund returns. Towards that end, we regress the net-of-fee monthly excess return (in excess of the risk free rate of interest) on a hedge fund index on the excess return on buy-and-hold and option-based risk factors in a multi-factor framework<sup>20</sup>. In particular, we estimate the following regression

$$R_t^i = c^i + \sum_{k=1}^K \mathbf{I}_k^i F_{kt} + u_t^i \quad (10)$$

where,

$R_t^i$  = net-of-fees excess returns on hedge fund index  $i$  during month  $t$ ,

$c^i$  = intercept for hedge fund index  $i$  over the regression period,

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believe that their straddle-like payoff will get captured by a combination of long call and long put options.

<sup>20</sup> As returns on option-based strategies have a larger order of magnitude compared to the buy-and-hold

$I_k^i$  = average factor loading of hedge fund index  $i$  on  $k^{th}$  factor during the regression period,

$F_{kt}$  = excess return on  $k^{th}$  factor during month  $t$ , ( $k=1, \dots, K$ ) where the factor could be a buy-and-hold or an option-based risk factor, and

$u_t^i$  = error term.

Given the lack of transparency and the large number of possible market and trading strategy combinations the hedge funds can follow, it is a challenging task to identify the dominant risk factors using limited data on their returns. This problem has been well recognized in the hedge fund literature. Researchers have addressed this problem by using a stepwise regression procedure either explicitly (Liang (1999), Fung and Hsieh (2000b)) or implicitly (Fung and Hsieh (2001a)) while identifying significant risk factors<sup>21</sup>. The stepwise regression involves adding and/or deleting variables sequentially depending on the F-value. One of the benefits of this procedure lies in its parsimonious selection of factors, while one of its shortcomings lies in the breakdown of standard statistical inference. The latter is a potential concern; however, it should only worsen the ability of the parsimoniously extracted factors to explain out-of-sample variation in hedge fund returns. Given that we obtain within-the-sample results that are consistent with other researchers and that we are able to replicate the out-of-sample performance of hedge funds, we believe that the benefits of using stepwise regression procedure outweigh its limitations.

### **3.1 Common Risk Exposures of Hedge Funds belonging to the HFR indexes**

We describe in Table 4 the factors that exhibit statistically significant relation in our step-wise regression procedure when the dependent variable is the returns on HFR's Event Driven, Event

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strategies, we scale them by a factor of hundred and use the scaled returns in our multi-factor model.

<sup>21</sup> Fung and Hsieh (2001a) search across five asset classes spanning twenty-six different markets and identify seven as the 'preferred habitats' of trend followers during extreme equity market movements (see

Arbitrage, Restructuring, Relative Value Arbitrage, Convertible Arbitrage, Equity Hedge, Equity Non-Hedge and Short Selling indexes<sup>22</sup>.

### **3.1.1 Significant Risk Exposures of HFR Event Arbitrage Index**

We find a non-linear risk-return tradeoff with the Event Arbitrage index showing significant factor loading on risk factor corresponding to writing at OTM put option on S&P 500 index (SPP<sub>o</sub>). This result is intuitive as Event Arbitrage strategy involves the risk of deal failure. A larger fraction of deals fail when markets are down and the Event Arbitrage strategy incurs losses. In contrast, when markets are up a larger proportion of deals go through and the strategy makes profits. But the profits are unrelated to the extent by which the market goes up. Thus, the payoff to Event Arbitrage strategy resembles that obtained by writing a naked put option on the market.

Fama-French's Size (SMB) factor shows significant relation suggesting that returns to Event Arbitrage strategies resemble those achieved by going long small stocks and short large stocks. This is intuitive as well, since the size of target firm is generally smaller than that of the acquiring firm. Going long the target's stock and short the acquirer's stock naturally results in a long exposure on Fama-French's Size factor. Fama-French's Value (HML) factor also shows significant relation suggesting a tilt towards value stocks. This would happen if the hedge funds were following Event Arbitrage strategy and the growth firms were trying to acquire value firms.

It is interesting to compare and contrast our analysis of the risks of Event Arbitrage strategy with Mitchell and Pulvino's (2001) findings of the risks of the same strategy<sup>23</sup>. They select 4750 merger events from 1963 to 1998 and examine the risks in a stock merger (by going long target's stock and going short the acquirer's stock) and those in a cash merger (by going long the target's

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their Table 5).

<sup>22</sup> We specify a five percent significance level for including an additional variable in our stepwise regression procedure. Tables 4 and 5 report the significant factors and the adjusted R-squares. We determine the significance using heteroskedasticity and autocorrelation consistent standard errors.



stock). They find that the risk of Merger or Event Arbitrage strategy resembles that of writing a naked put option on the market and having a long exposure to Fama-French's Size (SMB) factor. Interestingly, our multi-factor model also selects writing a put option on S&P 500 index and going long Fama-French's Size factor as dominant risk factors. These striking similarities suggest that our approach is able to capture dominant risk exposures of hedge funds following Event Arbitrage strategy.

### **3.1.2 Significant Risk Exposures of HFR Restructuring Index**

Restructuring strategy involves investing in the securities of firms in financial distress (i.e., firms that have filed for Chapter 11 or are undergoing some form of reorganization). For this strategy, similar to the Event Arbitrage index, we find a non-linear risk-return tradeoff. In particular, it shows a significant factor loading on risk factor corresponding to writing at OTM put option on S&P 500 index (SPP<sub>o</sub>). This result is intuitive as the probability of firms emerging from financial distress is lower when the markets are down due to firms losing business during market downturns. Thus, the payoff to this strategy resembles that obtained by writing a put option on the market.

In addition, we find Fama-French's Size (SMB) factor showing a significant relation with the Restructuring index. This is not surprising because smaller firms are more likely to be in distress. Further, we find that the Fama-French's Value (HML) factor also shows a significant relation. This is again consistent with the high book-to-market ratio firms being more likely to be in distress.

Typically, these securities are illiquid and infrequently traded. Our finding of a significant factor loading on lagged Russell 3000 index and Lehman High Yield index is consistent with this notion. Restructuring index also shows a significant factor loading on FRB Competitiveness-Weighted Dollar index and MSCI Emerging Market index. This may be due to the managers investing in distressed firms from emerging markets or those exposed to emerging markets.

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<sup>23</sup> Also see Baker and Savasoglu (2001) for analysis of 2088 cash and stock merger deals.

### ***3.1.3 Significant Risk Exposures of HFR Event Driven Index***

Similar to the Event Arbitrage and Restructuring indexes, we find a non-linear risk-return tradeoff in case of Event Driven index. This is manifested through a short position in an OTM put option on S&P 500 index ( $SPP_o$ ). Event Driven strategy involves taking bets on events such as mergers, takeovers and reorganizations. The risk in this strategy pertains to the non-realization of such events. This is more likely to happen during market downturns. The short position in put option is consistent with this economic interpretation.

We also find a positive loading on Fama-French's Size (SMB) and Value (HML) factor, Russell 3000 and lagged Russell 3000 indexes. As Event Driven strategy is similar to Event Arbitrage and Restructuring strategies, we find the risk factors to be similar and existing for similar reasons as mentioned before.

### ***3.1.4 Significant Risk Exposures of HFR Relative Value Arbitrage Index***

Relative Value Arbitrage strategy attempts to take advantage of relative pricing discrepancies between instruments like equities, debt, and derivative securities. As in the previous cases, we find that it also exhibits non-linear risk-return relation with the equity market index. The Relative Value Arbitrage index payoff resembles that from a short position in an OTM put option on the S&P 500 index ( $SPP_o$ ) suggesting that these strategies lose money during large down moves in equity market. Carhart's momentum factor also comes out significant with a negative factor loading suggesting that Relative Value Arbitrage funds follow a "contrarian" strategy. This finding is intuitive. Hedge funds employing such strategies follow securities with similar fundamental value and, when their prices diverge, then they buy under-valued securities (losers) and sell the over-valued securities (winners). This is opposite of what the momentum traders do, namely, buy winners and sell losers. As before, we also find Fama-French's Size (SMB) and Value (HML) factors coming out significant. This finding is consistent with the results of Gatev et al. (1999),

who replicate returns of Pairs Trading strategy, which is one of the strategies followed by Relative Value Arbitrage funds<sup>24</sup>.

### **3.1.5 Significant Risk Exposures of HFR Convertible Arbitrage Index**

Convertible Arbitrage strategy attempts to take advantage of relative pricing discrepancies between the theoretical and market price of convertible bonds. If a convertible bond appears to be undervalued, then the manager may purchase the bond and hedge out some of the risk components such as equity risk, credit risk and interest rate risk. As in the previous cases, we find that it also exhibits non-linear risk-return relation with the equity market index. The Convertible Arbitrage index payoff resembles that from a short position in an ATM put option on the S&P 500 index ( $SPP_a$ ) suggesting that these strategies lose money during large down moves in equity market. Lagged Russell index also comes out significant suggesting illiquid and infrequent trading nature of the bonds. Similar to Restructuring and Event Driven Indexes, we find that Convertible Arbitrage index also shows significant loading on Fama-French's Size (SMB) index and MSCI Emerging Market index.

### **3.1.6 Significant Risk Exposures of HFR Equity Hedge and Equity Non-Hedge indexes**

The HFR Equity Hedge index covers the original Long-Short strategy followed by Albert Winslow Jones in 1949. HFR include funds that follow long-short strategies into Equity Hedge and Equity Non-Hedge categories<sup>25</sup>. Hedge funds that aim to have relatively low net long exposure are included in HFR Equity Hedge index, while those with relatively high net long exposure are included in HFR Equity Non-Hedge index. This is confirmed by their betas with respect to Russell

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<sup>24</sup> Another potential strategy Relative Value Arbitrage funds may follow is analyzed by Mitchell, Pulvino and Stafford (2002) who provide an interesting analysis of the impediments to exploiting apparent arbitrage opportunities in equity markets when the market value of the company is less than the sum of its publicly traded parts. Although these situations suggest clear arbitrage opportunities, they show how the risks and market frictions preclude the prices from converging to fundamental value.

<sup>25</sup> See Schneeweis and Spurgin (2000) for the analysis of two mutual funds employing hedged equity

3000 index with Equity Hedge (Equity Non-Hedge) index showing a beta of 0.41 (0.75). Both the indexes show long exposure to Fama-French's Size (SMB) factor. This finding is intuitive, as one would expect the small stock universe to be less researched and therefore one has higher probability of finding mispriced stocks. A long exposure to SMB factor suggests that these managers buy undervalued small stocks and offset the market risk by going short on the large stocks<sup>26</sup>. Interestingly, Equity Hedge index shows negative factor loading on Fama-French's Value (HML) factor suggesting that the managers were long growth stocks during our sample period. This is not surprising as growth stocks outperformed value stocks during this period. Finally, Equity Hedge index also shows some exposure to commodities while the Equity Non-Hedge index shows some exposure to MSCI Emerging Markets.

### ***3.1.7 Significant Risk Exposures of HFR Short Selling index***

Short Selling strategy involves selling short overvalued securities with the hope of repurchasing them at lower prices in the future. Therefore, one expects their factor loadings to be opposite in sign to those for managers using long positions, such as Equity Hedge and Equity Non-Hedge. Our findings of negative betas on the market (Russell 3000 index), Fama-French's Size (SMB) factors and positive beta on Fama-French's Value (HML) factor are in line with this expectation. Finally, Short Selling Index shows a payoff that resembles a short position in an OTM call option on Russell 3000 index. This is again opposite to the short position in an OTM put option that we find in the other strategies, which are long the market. Negative beta on Russell 3000 index along with this short position in OTM call option suggests that Short Selling managers lose a lot during extremely bullish equity markets.

### ***3.1.8 Summary of Significant Risk Exposures of HFR Hedge Fund indexes***

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strategies.

<sup>26</sup> This can be achieved either through direct shorting of large stocks or through a short position in futures contract such as S&P 500 index that consists of large stocks.

Overall, the evidence indicates that most hedge fund strategies exhibit non-linear risk-return relation as manifested through significant betas on option-based risk factors. In particular, the payoffs of Event Arbitrage, Restructuring, Event Driven, Relative Value Arbitrage and Convertible Arbitrage strategies resemble that from writing a put option on the market index. This may be because these strategies relate to economic activity and lose money during large down moves in the equity market, or it may be because the managers, in order to improve their Sharpe ratio or to respond to their incentive contract, create (either directly or indirectly through dynamic trading) a payoff similar to that from writing a put option<sup>27</sup>. Further, the risk exposures of Event Arbitrage and Relative Value Arbitrage estimated using our approach are consistent with the findings of Mitchell and Pulvino (2001) and Gatev et al. (1999) who use detailed replication methodology to estimate the risk of these strategies.

### **3.2 Robustness checks**

Before proceeding further, we examine the robustness of our results in terms of the choice of database used and the choice of alternative strike prices for the construction of option-based factors.

#### ***3.2.1 Choice of database***

Previous research on hedge funds has used HFR, CSFB/Tremont (TASS) and Zurich/MAR databases. One obvious question is how sensitive are the findings to the choice of database. To answer this question, we repeat our analysis using CSFB/Tremont indexes<sup>28</sup>. The

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<sup>27</sup> It is well known that by writing put options managers can improve their Sharpe ratios. See, Goetzmann et al. (2001) for option writing strategies that maximize the Sharpe ratio and Lo (2001) for returns to a hypothetical hedge fund that writes OTM put options on the S&P 500 index. Siegmann and Lucas (2002) examine the trading behavior of a hedge fund manager minimizing expected shortfall below a certain level. They find that having a payoff similar to the short position in a put option is an optimal response in certain circumstances.

<sup>28</sup> Zurich/MAR hedge fund indexes represent returns earned by a median hedge fund belonging to a particular strategy. The median fund changes from month to month, and therefore, Zurich/MAR index returns represent rewards for bearing different combinations of systematic risks each month. Our

choice of index can potentially affect the results due to reasons such as extent of coverage, the method of index construction (e.g. equal weighting by HFR vis-à-vis value weighting by CSFB/Tremont), etc. We select four CSFB/Tremont strategies that are common with HFR, namely, Event Driven, Convertible Arbitrage, Long/Short Equity (Equity Hedge in case of HFR) and Dedicated Short-Bias (Short Selling in case of HFR). We report the results from regression in equation (10) in Table 4.

Similar to HFR's Event Driven Index, CSFB/Tremont's Event Driven index shows significant non-linearity. In particular, its payoff resembles that from writing an OTM put option on S&P 500 index. It also shows positive loading on Fama-French's Size (SMB) and MSCI Emerging Market factors. For CSFB/Tremont's Convertible Arbitrage strategy, we find exposures to lagged Russell 3000 index and the Lehman High Yield index suggesting the illiquid nature of the bonds and the credit risk involved in the strategy. For CSFB/Tremont's Long/Short Equity strategy, we find exposures that are very similar to those of HFR's Equity Hedge and Equity Non-Hedge indexes. In particular, we find long exposure on Russell 3000 index and Fama-French's Size (SMB) and a short exposure to Fama-French's Value (HML) factor. As expected, CSFB/Tremont's Dedicated Short-Bias strategy shows negative loading on Russell 3000 index and Fama-French's Size (SMB) and a positive loading on and Fama-French's Value (HML) factor. These exposures are similar to those of HFR's Short Selling index. Overall, both HFR and CSFB/Tremont indexes exhibit similar risk exposures that are consistent with the types of trading strategies the hedge fund claim to follow.

### ***3.2.2 Choice of Option Strike Prices***

As we find that a large number of hedge funds exhibit exposure similar to writing a put option on the market, it suggests that they bear significant tail risk. Hence, we examine the

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methodology can be applied to Zurich/MAR index by constructing an equally weighted index of hedge funds belonging to different strategies.

robustness of our results by capturing even higher tail risk by specifying option-based strategies using deeper out-of-the-money options. In particular, we specify four different degrees of moneyness ranging from half a standard deviation to two standard deviations, where the standard deviation is computed using daily returns from the month immediately preceding the one for which option returns are calculated. We observe that when one moves too far away from the at-the-money options, the contracts become illiquid and the prices become less reliable. We exercise caution by removing the outliers corresponding to the deeper out-of-the money options and find results that are qualitatively similar.

The fact that the Size factor turns out to be significant for a number of hedge fund strategies indicates that they invest in small stocks. It is possible that due to dynamic trading, the risk-return relationship with respect to small stocks may be nonlinear; in which case, options on S&P 500 Composite index may not be able to capture this effect. Therefore, we examine the robustness of our findings using options on Russell 2000 index traded on the Chicago Mercantile Exchange. Unfortunately, these contracts are highly illiquid and at times, we are unable to find reasonable prices. However, for the period during which we observe reliable prices, we find results similar to those obtained with options on the S&P 500 Composite index.

Finally, instead of using European-style options, we repeat our analysis with American-style three-month-to-maturity options on S&P 500 futures contracts and, once again, find qualitatively similar results<sup>29</sup>. This suggests that our findings are robust to the inclusion of deeper out-of-the money options, to the choice of a broader equity index and to the consideration of American-style options.

This concludes our discussion of the in-sample analysis of risk exposures of hedge funds. We now proceed to examine how well the in-sample risk exposures capture the out-of-sample performance of hedge funds.

#### 4. Out-of-Sample Analysis of Hedge fund Risk Exposures

If the risk exposures reported in Tables 4 and 5 are mere statistical artifacts of data, then these are unlikely to track hedge fund returns in an out-of-sample analysis. However, if they represent the true economic risks of different hedge fund strategies, then the replicating portfolios based on these factor loadings should do a good job of mimicking the out-of-sample performance of hedge funds. We examine this issue by constructing a replicating portfolio for each of the HFR and CSFB/Tremont indexes using the factor loadings obtained from our multi-factor model. We compute the difference between the monthly return on hedge fund index and that on the respective replicating portfolio. We conduct standard t-test and Wilcoxon sign-test to examine if the differences in the mean and median returns on the index and its respective replicating portfolio are statistically significant. We report the results in Table 6. We find the mean and median differences between the HFR and CSFB/Tremont indexes, and their replicating portfolios are statistically insignificant using both the t-test and the Wilcoxon sign-test, the only exception being CSFB/Tremont's convertible arbitrage index<sup>30</sup>. Figure 1 graphically illustrates the returns on HFR indexes and those on the replicating portfolios during July 2000 to December 2001 period. It shows that the portfolios based on significant risk exposures estimated through our model closely track the hedge fund returns during out-of-sample period. This suggests that our approach is able to capture the dominant economic risk exposures of hedge funds<sup>31</sup>.

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<sup>29</sup> We are grateful to David Hsieh for providing data on American style options for this robustness check.

<sup>30</sup> In general, the difference in the mean returns between the hedge fund indexes and the replicating portfolios from model is about 24 basis points for the HFR indexes and about 94 basis points for the CSFB/Tremont indexes. Although this difference is not statistically significant in all except one case, it is nevertheless economically significant. A part of this difference can be attributed to survivorship and other biases (see Fung and Hsieh (2000a)). The rest may be a compensation for bearing risks not captured by our model.

<sup>31</sup> Since investors invest in individual hedge funds, we repeat the out-of-sample analysis with individual hedge fund returns and report the findings in Appendix A.



A wide range of hedge fund strategies exhibiting nonlinear payoffs has important implications for portfolio decisions involving hedge funds. We investigate this issue in the following section.

## 5. Portfolio Decisions with Hedge Funds

Our results from Section 3 show that the payoffs on a wide range of hedge fund indexes resemble those from selling out-of-the-money put options on the market index. This suggests that these hedge funds may be selling portfolio insurance, a strategy providing positive returns when the market does not lose much and experiencing large losses in extreme down market conditions. Hedge funds market themselves as absolute return vehicles, which aim to deliver positive returns irrespective of the market conditions. Arguably, hedge fund investors care about absolute value of losses (and not losses relative to a benchmark index)<sup>32</sup>. Therefore, a portfolio construction framework involving hedge funds must explicitly account for large losses (i.e., the tail risk of hedge funds) in down market conditions. Fung and Hsieh (1999b) argue that asset allocation involving hedge funds should not be based on the mean-variance (M-V) framework as it is appropriate only for normally distributed returns or for quadratic preferences of the investors. They show that although the rankings based on the mean-variance criterion are approximately correct, risk assessment and management based on such a criterion will not be correct as it does not take into account the probability of large negative returns. Our results from Section 3 show that hedge fund payoffs are nonlinear and asymmetric with significant negative tail risk. Therefore, any portfolio constructed involving hedge funds needs to explicitly account for their tail risk, an important issue that we address in this section of our paper.

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<sup>32</sup> One of the widely used performance measures in the hedge fund industry is the “drawdown” that measures the distance between the peak to trough using cumulative return history of hedge fund returns.

The Basle Committee on Banking Supervision has recommended use of risk management framework such as VaR to better understand and manage the downside risk. Hull (2000, page 342) reports a number of financial institutions, corporate treasurers and fund managers use VaR. However, researchers such as Artzner et al (1999) have shown that VaR has problematic properties and have proposed the use of Conditional Value-at-Risk (CVaR), also known as Conditional Expected Loss (CEL)<sup>33</sup>. Some of these problems are the failure to satisfy the sub-additivity property, i.e., VaR of a portfolio of securities may be larger than the sum of VaR of each of the securities in the portfolio. Further, VaR function is non-convex and non-differentiable. VaR measures the maximum loss for a given confidence level over a given period of time<sup>34</sup>. In contrast, CVaR corresponds to the expected loss conditional on the losses being greater than or equal to the VaR. It equals the statistical mean of the losses exceeding the VaR. While the VaR focuses only on the frequency of extreme events, CVaR focuses on both frequency and size of losses in case of extreme events.

## 5.1 Theoretical Framework for VaR and CVaR

In this section, we define the concepts of VaR and CVaR by evaluating the risk beyond the VaR using simple statistics. Let the return on a portfolio over a given period of time is denoted by  $R$ . Let the probability density function (PDF) of  $R$  be denoted by  $f_R$  and the cumulative distribution function (CDF) denoted by  $F_R$ . We denote the VaR of the portfolio for a probability level  $p$  as  $\text{VaR}(F_R, p)$  in order to indicate its dependence on the CDF and the specified probability

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<sup>33</sup> Basak and Shapiro (2001) use a measure closely related to CEL, namely, Limited Expected Losses (LEL) and show that using such a constraint for investor leads to optimal strategies for which magnitude of extreme losses is kept under control.

<sup>34</sup> For example, for a probability level of 99%, VaR equal to \$1 million implies that the loss should not exceed \$1 million in 99 cases out of 100 on average.

level. When expressed as a percentage of initial value of the portfolio and as a positive number, the VaR of the portfolio can be expressed as

$$VaR(F_R, p) = -F_R^{-1}(1 - p) \quad (11)$$

The CVaR measures the expectation of the losses greater than or equal to the VaR and is given by<sup>35</sup>

$$\begin{aligned} CVaR(F_R, p) &= -E(R | R \leq -VaR) \\ &= -\frac{\int_{-\infty}^{-VaR} z f_R(z) dz}{F_R(-VaR)} \end{aligned} \quad (12)$$

Considering the various advantages of CVaR over VaR, we use CVaR as a risk management tool to control the tail risk of a portfolio involving hedge funds. While optimizing, one can either impose a distributional assumption on the security returns or use the empirical distribution of security returns. Since CVaR focuses on the tail risk, considering parameterized distributions may not be able to fully capture this risk due to their potentially poor tail properties. Therefore, we use the empirical distribution of hedge fund returns for Mean-CVaR optimization<sup>36</sup>.

## 5.2 Mean-Variance and Mean-CVaR optimization results

As the M-V framework implicitly assumes normality of asset returns, it is likely to underestimate the tail risk for assets with negatively skewed payoffs. In this section, we test this conjecture by using the M-CVaR framework theorized above. Specifically, we compare the tail losses on M-V optimal portfolios with those on the M-CVaR optimal portfolios for different confidence levels. In particular, we construct a M-V efficient frontier and a M-CVaR efficient frontier using the eight HFR hedge fund strategies. We compute the CVaRs of the M-V efficient

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<sup>35</sup> Notice that the numerator measures the size of the losses beyond the VaR while the denominator measures the frequency of losses greater than or equal to the VaR.

<sup>36</sup> We follow Palmquist et al (1999) and Alexander and Baptista (2002b) to construct the Mean-CVaR frontier. It turns out to be a linear programming problem which we solve using MATLAB's *linprog* function. The details of formulating the mean-CVaR optimization problem as a linear programming problem are given in

portfolios of different volatilities and compare them with those of M-CVaR efficient portfolios with volatilities. We also measure the differences in their mean returns, which indicate how much of the return one has to give up for reducing the tail-risk.

Table 9 reports the CVaRs of M-V and M-CVaR efficient portfolios at 90%, 95% and 99% confidence levels. It also reports ratios of the CVaRs and differences in mean returns of the two portfolios. As expected, CVaR increases with the portfolio volatility and confidence level (due to going out further in the left tail at higher confidence level). The average ratio of CVaR of M-V and M-CVaR portfolio ranges from 1.12 at 90% confidence level to 1.54 at 99% confidence level. This suggests that tail risk is significantly underestimated using the M-V approach, the range of underestimation being 12% to 54% for confidence level ranging from 90% to 99%<sup>37</sup>. Figure 3 illustrates how the ratio of CVaR of M-V efficient portfolio to the CVaR of a M-CVaR efficient portfolio of hedge funds varies with the portfolio volatility. As mentioned earlier, it is clear from the figure that the ratio is higher for higher confidence level. However, the ratio decreases with increasing portfolio volatility, suggesting that for efficient portfolios of high volatility, the underestimation of loss due to use of M-V approach is less<sup>38</sup>. In general, the M-V approach underestimates the loss compared to the M-CVaR approach, and this underestimation is substantial for portfolios with low volatility. The differences in mean returns reported in Table 9, which can be thought of as the price investors pay to reduce tail-risk, are consistent with this, they are higher for portfolios with low volatility. For 90% and 95% confidence levels, the difference in mean returns is up to 7 basis points while at 99% confidence level it is up to 17 basis points.

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Appendix B.

<sup>37</sup> This is economically significant number considering that if a hedge fund is managing \$1 billion, if the CVaR of M-CVaR efficient portfolio is 1% at 99% confidence level, the average loss can exceed \$10 million in 1 out of 100 cases while using a M-V approach the average loss can exceed \$15.4 million at the same confidence level.

<sup>38</sup> This result seems to be consistent with Alexander and Baptista (2002b) who find that the mean-variance efficient portfolios with smaller standard deviations may not be efficient in the mean-conditional expected loss (CEL) space. As mentioned earlier, their CEL measure is equivalent to our CVaR measure.

Having compared and contrasted the differences between efficient portfolios constructed using M-V and M-CVaR approach, we now proceed with the examination of long-run risk return tradeoffs of hedge funds.

## **6. Long-run Performance of Hedge funds**

One of the limitations investors face while dealing with hedge funds is that the return history of hedge fund indexes goes back at most to January 1990. One way to circumvent this limitation is to work with the underlying risk factors for which longer return history is available. For example, data on market, size, value and momentum factors is available from 1927. For the option-based factors, although returns data is available only from 1982, it is possible to construct a theoretical return series going back to 1927 using Black and Scholes' (1973) formula<sup>39</sup>. This provides us with the return history of key risk factors going back to 1927. In order to shed light on the long-run performance, we regress the hedge fund index returns on market, size, value, momentum and option-based risk factors and re-estimate the factor loadings. Using these factor loadings, we re-compute the returns of the hedge fund index replicating portfolios from January 1927 to December 1989. We call these the long-run systematic returns of different hedge fund strategies. In order to compare returns on a like to like basis, we also re-compute systematic returns to the indexes during the recent period (January 1990 to June 2000) using the simplified model. We report the summary statistics of these returns for the HFR indexes in Table 10.

We find interesting differences between the recent returns and long-run systematic returns. For the HFR indexes, the mean long-run (recent) monthly return varies from 0.0 (0.15) percent for Short Selling strategy to 0.97 (1.26) percent for Restructuring strategy. The corresponding volatility ranges from 1.45 (0.88) percent for Event Arbitrage strategy to 6.27 (5.81) percent for

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<sup>39</sup> We use historical volatility (based on five-year rolling window) to compute the option prices. For the first five years, we use average volatility during the five-year period. We compute returns based on theoretical

Short Selling strategy. The magnitude of long-run CVaRs at 90%, 95% and 99% levels across the eight HFR indexes are higher on average by 100%, 60% and 40% respectively than the corresponding recent period CVaRs. The findings with the CSFB/Tremont indexes are similar as well (see Table 11). For the CSFB/Tremont indexes, the mean long-run (recent) monthly return varies from -0.18 (-0.55) percent for Short Selling strategy to 0.83 (1.26) percent for Event Driven strategy. The corresponding volatility ranges from 1.00 (0.68) percent for Convertible Arbitrage strategy to 6.65 (4.88) percent for Short Selling strategy. The magnitude of long-run CVaRs at 90%, 95% and 99% levels are higher on average by 90%, 70% and 100% respectively than the corresponding recent period CVaRs. Overall, across all the indexes, we find that the long-run returns are smaller, the long-run volatilities are larger and the magnitude of long-run CVaRs are larger compared to the recent period.

In order to examine whether the differences in the long-run returns and volatilities are statistically significant from those in the recent period, we conduct the standard t-test (for the means), Wilcoxon sign test (for the median) and variance ratio test (for standard deviations). We report the findings in Table 12. For all the HFR indexes, we find that the mean long-run returns are smaller than those for the recent period by about 23 basis points per month (or 2.76 percent per annum) and this difference is statistically significant in three cases<sup>40</sup>. The long-run median returns are also smaller than those during the recent period by about 25 basis points (or 3.00 percent per annum) and the difference is statistically significant for three indexes. The long-run volatilities are also significantly larger than those in the recent period in seven out of eight cases. The results for the CSFB/Tremont indexes are qualitatively similar. For all strategies except short-selling, the long-run mean and median returns are smaller than those during the recent period, and the difference is statistically significant in case of two indexes for mean returns and one index for

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prices for 1927-1982 and based on market prices for the remaining period.

<sup>40</sup> Even in cases where the difference is not statistically significant, a figure ranging from 2.5% to 3.00% per

median returns. The long-run volatilities are also significantly larger than those in the recent period in three out of the four cases<sup>41</sup>. Overall, these findings suggest that the performance of hedge funds during the recent period appears significantly better compared with their long-run performance.

## 7. Concluding Remarks

In this paper, we characterize the linear and non-linear risks of a wide range of hedge fund strategies using buy-and-hold and option-based risk factors. For this purpose, we employ a two-step approach. In the first step, we estimate the factor loadings of hedge funds using the returns on standard asset classes and options on them as factors. We construct replicating portfolios that best explain the in-sample variation in hedge fund index returns. In the second step, we examine how well these replicating portfolios capture the out-of-sample performance of hedge funds. We conduct the analysis both at the index level as well as at an individual level.

We have four main results. First, we find that it is important to allow for non-linear risk-return relation while analyzing hedge funds. Along with the non-linear exposure to equity market index, we find that hedge funds also exhibit significant risk exposures to Fama-French's (1993) size and value factors and Carhart's (1997) momentum factor. Second, we observe that a wide range of hedge fund strategies exhibit returns similar to those from writing a put option on the equity index. The observed non-linearities across multiple strategies suggest that these events are not statistical outliers, but represent important risks borne by hedge fund investors. Third, since hedge funds

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annum is economically significant.

<sup>41</sup> In order to make the HFR results comparable with those from CSFB/Tremont, we divide the HFR sample period (Jan 90 - June 00) into two sub-periods, Jan 90 - Dec 93 and Jan 94 - June 00, for the second sub-period to coincide with that of CSFB/Tremont. We find that the difference in the mean and median returns over the long-run and those during the second sub-period to be 20 and 21 basis points, figures comparable to the 23 and 25 basis points we find using Jan 90 - Jun 00 period. Also, the magnitude of CVaRs during the second sub-period compared to that during the long-run are 100%, 70% and 40% lower, figures comparable to 100%, 60% and 40%, we find using the Jan 90 - Jun 00 period.

exhibit significant left-tail risk, we compare and contrast the tail losses of portfolios constructed using mean-variance framework and mean-conditional value-at-risk framework. We find that using the traditional mean-variance framework, substantially underestimates the tail losses and this underestimation is most severe for portfolios with low volatility. Finally, we compare and contrast the long-run systematic returns of hedge funds with those observed during recent period. Almost across all hedge fund indexes, we find that the long-run returns are lower, the long-run volatilities are higher and the long-run tail losses are larger compared to those during the recent period.

Understanding the risk exposures of hedge funds is an important area of research. We need a better understanding of this issue while making investment management decisions involving hedge funds. Unfortunately, this is a tricky issue as hedge funds provide limited disclosure. In this context, our approach provides useful information to investors dealing with portfolio construction and risk management related issues. At a more general level, it indicates whether a fund has been classified correctly or not and, when applied on an ongoing basis, it enables investors to address issues like hedge fund style drift. Estimation of hedge fund risks is also important as a large number of hedge funds propose risk-free rate as a benchmark for claiming incentive fees. This would be appropriate only if they carried no systematic risks. However, we find that a large majority of hedge funds carry significant amount of systematic risk. We believe our findings raise important concerns relating to issues like benchmark design and manager compensation<sup>42</sup>. In addition, our analysis provides a tool to measure the net and gross risk exposures of hedge funds. This can help address regulators' concern regarding the potential risk hedge funds can pose to stability of financial markets.

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<sup>42</sup> Previous researchers including Brown et al (1999) and Agarwal and Naik (2000), examining persistence in hedge fund managers' performance have used peer group average as a benchmark to adjust for systematic risk. It would be interesting to examine persistence in performance after adjusting for systematic risk using our model.



Popular press classifies some hedge fund strategies as short-volatility strategies. The short positions in put options that we find are consistent with this notion. If one can locate or construct an instrument whose payoff is directly related to volatility of financial markets, then it would be interesting to include it as an additional asset class factor. Similarly, it would also be interesting to create proxies that capture returns from arbitrage opportunities. For example, one could use a statistical arbitrage model and compute returns to arbitraging mispriced securities. Returns to such strategies can also be used as additional factors in our model to capture some of the active (i.e. non-systematic) risk of hedge funds. These issues are a part of our ongoing research agenda.

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## Appendix A: Out-of-Sample Analysis with Individual Hedge Funds

Our analysis in Section 4 is at the hedge fund index level. Since investors invest in individual hedge funds, we also examine how well our replicating portfolios are able to explain the out-of-sample variation in individual hedge funds compared to the hedge fund indexes themselves. Towards that end, we regress the returns of individual hedge funds belonging to the different indexes on our replicating portfolios for those indexes during July 2000 to August 2001 period<sup>43</sup>. We report in Table 7 the distribution of adjusted R-squares obtained with our HFR and CSFB/Tremont index replicating portfolios. In order to compare how well our index replicating portfolios are able to explain the out-of-sample variation in individual hedge fund returns, we need to know how well the hedge fund indexes to which they belong explain their returns in the first place. For this purpose, we also regress the returns of individual hedge funds on the respective HFR and CSFB/Tremont indexes. We report in Table 8 the distribution of adjusted R-squares of these regressions. In Figure 2, we plot the histogram of adjusted R-squares from the regressions using HFR and CSFB/Tremont replicating portfolios and indexes.

As can be seen from Table 7, our replicating portfolios exhibit mean (median) adjusted R-squares ranging from 0.3% to 60.9% (-5.0% to 61.0%) for HFR and 23.8% to 67.9% (18.1% to 81.1%) for CSFB/Tremont funds. This range of mean and median adjusted R-squares is similar to those obtained using the respective HFR and CSFB/Tremont hedge fund indexes. As shown in Table 8, indexes exhibit mean (median) adjusted R-squares ranging from 16.1% to 68.6% (8.4% to 66.4%) for HFR and 21.4% to 59.8% (11.2% to 75.4%) for CSFB/Tremont funds. Overall, the replicating portfolios explain an average of 26.7% (median of 22.5%) variation in out-of-sample returns of individual HFR funds and an average of 27.2% (median of 22.6%) variation in the out-of-sample returns of individual CSFB/Tremont funds. The corresponding figures for the indexes

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<sup>43</sup> We only consider those individual hedge funds that have at least 6 monthly returns during July 2000 to June 2001 period. For the CSFB/Tremont database, individual funds following “Long/Short Equity” strategy

are mean (median) adjusted R-squares of 30.9% (27.4%) for HFR and 23.0% (13.8%) for CSFB/Tremont. These figures are very much comparable to those we obtain using replicating portfolios. In fact, for CSFB/Tremont, overall, our replicating portfolios do a slightly better job than the indexes in explaining the variation in out-of-sample returns of individual funds. There can be two reasons why our replicating portfolios better explain the out-of-sample variation in individual CSFB/Tremont funds. First, CSFB/Tremont indexes are constructed using a subset of funds and are weighted by assets under management. As a result, they give higher weight to larger funds. In contrast, our analysis of individual funds includes all funds and the mean adjusted R-square is based on an equally weighted average of all funds. Second, the composition of the CSFB/Tremont indexes may change during the out-of-sample (i.e. post June 2000) period while the composition of the index replicating portfolios remains the same. These two reasons may lead to the CSFB/Tremont indexes explaining a smaller proportion of out-of-sample variation in individual hedge funds.

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are classified under “Long/Short Equity Hedge” category.

## Appendix B: Formulation of Mean-CVaR optimization problem

For any given month, let the hedge fund index returns be denoted by  $\xi = (\xi^1, \xi^2, \xi^3, \dots, \xi^N)$  where  $i = 1, 2, \dots, N$  denote the  $N$  hedge fund indexes. If the investor has a budget of 1 unit and his positions in the different hedge fund indexes are  $\omega = (\omega_1, \omega_2, \dots, \omega_N)$  such that  $\omega_i \geq 0$  (short sales constraint) and  $\sum_{i=1}^N \omega_i = 1$  (budget constraint), then the value of his portfolio is  $W = \omega^T \xi = \sum_{i=1}^N \omega_i \xi_i$ .

CVaR has mathematical properties that make it amenable as a risk measure. In particular it is convex which makes it possible to construct efficient algorithms for controlling and, in particular, minimizing CVaR. It is shown in Rockafellar and Uryasev (2000) that the minimization of the CVaR function can be reduced to the minimization of the function,

$$F_p(\mathbf{w}, \mathbf{z}) = \mathbf{z} + \frac{1}{1-p} E \left\{ [f(\mathbf{w}, \mathbf{x}) - \mathbf{z}]^+ \right\}. \quad (\text{A1})$$

The function  $F_p(\mathbf{w}, \mathbf{z})$  is convex with respect to  $\mathbf{z}$ , the threshold loss. It is also convex with respect to  $\mathbf{w}$  if the function  $f(\mathbf{w}, \mathbf{x})$  is convex with respect to  $\mathbf{w}$ . Minimizing  $F_p(\mathbf{w}, \mathbf{z})$  simultaneously finds VaR and the minimal CVaR value. The minimum of CVaR is the minimum of  $F_p(\mathbf{w}, \mathbf{z})$ , the optimal portfolio weights  $\mathbf{w}$  equal the optimal decision vector, and the equivalent portfolio VaR is the smallest of optimal  $\mathbf{z}$ .

The function  $F_p(\mathbf{w}, \mathbf{z})$  can be calculated using various approaches. If the expectation can be calculated or approximated analytically, then to optimize the function  $F_p(\mathbf{w}, \mathbf{z})$  one can use non-linear programming techniques. However, we can approximate the expectation in the function  $F_p(\mathbf{w}, \mathbf{z})$  using equally probable scenarios,  $\mathbf{x}_j$ ,  $j = 1, \dots, J$  i.e.

$$E\{[f(\mathbf{w}, \mathbf{x}) - \mathbf{z}]^+\} = \frac{1}{J} \sum_{j=1}^J [f(\mathbf{w}, \mathbf{x}_j) - \mathbf{z}]^+ \quad (\text{A2})$$

If both the loss function  $f(\mathbf{w}, \mathbf{x}_j)$  and the feasible set  $\Omega$  are convex, then to minimize CVaR we can solve the following *convex* optimization problem:

$$\min_{\mathbf{w} \in X, \mathbf{V} \in R} F_a(\mathbf{w}, \mathbf{z})$$

where,

$$F_a(\mathbf{w}, \mathbf{z}) = \mathbf{z} + \mathbf{n} \sum_{j=1}^J [f(\mathbf{w}, \mathbf{x}_j) - \mathbf{z}]^+$$

and the constant  $\mathbf{n} = \frac{1}{(1-p)J}$ .

In addition, if the loss function  $f(\mathbf{w}, \mathbf{x}_j)$  is linear with respect to  $\mathbf{w}$  and the set  $\Omega$  is given by linear inequalities, then we can reduce the optimization problem to the following linear programming problem:

$$\min_{\mathbf{w} \in R^N, \mathbf{z} \in R^J, \mathbf{V} \in R} \mathbf{V} + \mathbf{n} \sum_{j=1}^J z_j$$

subject to

$$\omega_i \geq 0$$

$$\sum_{i=1}^N \omega_i = 1$$

$$z_j \geq f(\mathbf{w}, \mathbf{x}_j) - \mathbf{V}, \quad z_j \geq 0, \quad j = 1, \dots, J,$$

where  $z_j, j = 1, \dots, J$  are dummy variables,

where  $J$  is the number of return observations,

$$\text{and } \mathbf{n} = \frac{1}{(1-p)J}$$



**Table 1: Summary Statistics**

This table shows the means, standard deviations (SD), medians, skewness (Skew), kurtosis, minimum and maximum of returns for eight HFR hedge fund indexes (Panel A), twelve buy-and-hold and four option-based risk factors (Panel B) during Jan 90 to June 00 and four CSFB/Tremont hedge fund indexes (Panel C) during Jan 94 to June 00.

<b>Panel A: HFR Hedge Fund Indexes</b>							
Hedge fund strategy	Mean	SD	Median	Skew	Kurtosis	Min.	Max.
<i><b>Non-Directional</b></i>							
Event Arbitrage	1.03	1.32	1.33	-3.24	17.18	-6.46	2.90
Restructuring	1.29	1.90	1.35	-0.81	8.88	-8.50	7.06
Event Driven	1.33	1.94	1.53	-1.62	9.42	-8.90	5.13
Relative Value Arbitrage	1.15	1.16	1.29	-1.26	13.31	-5.80	5.72
Convertible Arbitrage	0.95	1.01	1.16	-1.48	6.30	-3.19	3.33
Equity Hedge	1.82	2.65	1.82	0.10	4.57	-7.65	10.88
<i><b>Directional</b></i>							
Equity Non-Hedge	1.71	4.06	2.28	-0.59	4.17	-13.34	10.74
Short Selling	0.07	6.40	-0.16	0.13	4.64	-21.21	22.84
<b>Panel B: Risk Factors</b>							
Risk Factor	Mean	SD	Median	Skew	Kurtosis	Min.	Max.
<i><b>Buy-and-Hold Risk Factors</b></i>							
<i><b>Equity</b></i>							
Russell 3000	1.39	3.94	1.69	-0.67	4.75	-15.32	11.22
MSCI World Excluding US	0.66	4.83	0.71	-0.18	3.49	-13.47	14.67
MSCI Emerging Markets	1.01	6.80	1.41	-0.64	5.49	-28.91	16.53
Fama-French SMB factor	-0.03	3.46	-0.08	0.54	6.15	-11.66	15.40
Fama-French HML factor	-0.31	4.16	-0.43	-1.14	9.73	-21.51	14.23
Momentum factor	0.94	4.18	1.17	-0.27	4.75	-11.47	13.77
<i><b>Bond</b></i>							
SB Government and Corporate Bond	0.63	1.25	0.77	-0.06	3.25	-2.37	4.65
SB World Government Bond	0.63	1.81	0.75	0.16	3.39	-3.63	6.11
Lehman High Yield	-0.10	3.16	0.05	-4.16	35.60	-25.47	10.16
Default Spread	-0.09	1.65	-0.21	0.06	3.36	-5.50	3.67
<i><b>Currency</b></i>							
FRB Competitiveness-Weighted Dollar	0.45	1.20	0.30	0.42	3.68	-2.78	3.96
<i><b>Commodity</b></i>							
Goldman Sachs Commodity	0.65	5.04	0.79	0.54	4.36	-12.28	18.52
<i><b>Option-based Risk Factors</b></i>							
S&P 500 At-the-Money Call	4.77	84.09	-17.01	0.76	2.80	-98.57	236.24
S&P 500 Out-of-the-Money Call	3.36	93.80	-23.69	1.04	3.53	-99.35	300.60
S&P 500 At-the-Money Put	-24.38	84.72	-57.04	2.20	8.77	-95.30	386.02
S&P 500 Out-of-the-Money Put	-27.30	91.49	-62.76	2.69	11.67	-95.80	422.34
<b>Panel C: CSFB/Tremont Hedge Fund Indexes</b>							
Hedge fund strategy	Mean	SD	Median	Skew	Kurtosis	Min.	Max.
<i><b>Non-Directional</b></i>							
Event Driven	1.00	1.97	1.26	-3.59	24.01	-11.77	3.68
Convertible Arbitrage	0.83	1.50	1.15	-1.59	6.62	-4.68	3.57
Long/Short Equity	1.41	3.68	1.36	-0.04	5.16	-11.43	13.01
<i><b>Directional</b></i>							
Dedicated Short-Bias	-0.26	5.26	-0.39	1.11	6.18	-8.69	22.71

**Table 2: Correlation between the Hedge Fund Indexes and Asset Class Factors**

This table shows the correlations between the eight HFR hedge fund indexes and the twelve buy-and-hold risk factors during our sample period (Jan 90 to June 00). The table also shows the correlation between the four CSFB/Tremont hedge fund indexes and the twelve risk factors during the entire sample period from Jan 94 and June 00. The buy-and-hold risk factors are Russell 3000 index (RUS), MSCI excluding the US index (MXUS), MSCI Emerging Markets index (MEM), Fama-French Size and Book-to-Market factors (SMB & HML), Momentum factor (MOM), Salomon Brothers Government and Corporate Bond index (SBG), Salomon Brothers World Government Bond index (SBW), Lehman High Yield Composite index (LHY), Federal Reserve Bank Competitiveness-Weighted Dollar index (FRBI), Goldman Sachs Commodity index (GSCI) and the change in the default spread in basis points (DEFSPR). The abbreviations for different hedge fund strategies are Event Arbitrage (EA), Restructuring (REST), Event Driven (ED), Relative Value Arbitrage (RVAL), Convertible Arbitrage (CA), Equity Hedge or Long/Short Equity (EH), Equity Non-Hedge (ENH) and Short Selling or Dedicated Short-Bias (SHORT). Correlations significant at the bonferroni-adjusted significance level of 5% are shown in bold face.

	HFR								CFSB/TREMONT			
	EA	REST	ED	RVAL	CA	EH	ENH	SHORT	ED	CA	EH	SHORT
<b>RUS</b>	<b>0.49</b>	<b>0.42</b>	<b>0.66</b>	<b>0.39</b>	<b>0.39</b>	<b>0.67</b>	<b>0.81</b>	<b>-0.71</b>	<b>0.61</b>	0.18	<b>0.68</b>	<b>-0.67</b>
<b>MXUS</b>	0.29	0.29	<b>0.43</b>	0.30	0.27	<b>0.45</b>	<b>0.52</b>	<b>-0.49</b>	<b>0.61</b>	0.13	<b>0.66</b>	<b>-0.64</b>
<b>MEM</b>	<b>0.36</b>	<b>0.54</b>	<b>0.58</b>	<b>0.41</b>	<b>0.39</b>	<b>0.54</b>	<b>0.63</b>	<b>-0.53</b>	<b>0.63</b>	0.23	<b>0.65</b>	<b>-0.61</b>
<b>SMB</b>	0.29	<b>0.48</b>	<b>0.49</b>	<b>0.38</b>	0.30	<b>0.56</b>	<b>0.57</b>	<b>-0.57</b>	<b>0.45</b>	0.20	<b>0.54</b>	<b>-0.49</b>
<b>HML</b>	-0.13	-0.12	-0.29	-0.05	-0.16	<b>-0.59</b>	<b>-0.57</b>	<b>0.68</b>	<b>-0.53</b>	-0.06	<b>-0.72</b>	<b>0.72</b>
<b>MOM</b>	-0.04	-0.22	-0.03	<b>-0.35</b>	-0.18	0.16	0.07	-0.14	0.12	-0.14	0.28	-0.18
<b>SBG</b>	0.14	0.05	0.15	0.04	0.20	0.15	0.17	-0.11	0.05	0.12	0.13	-0.06
<b>SBW</b>	-0.03	-0.20	-0.10	-0.15	-0.05	0.00	0.01	-0.05	-0.11	-0.27	0.00	0.04
<b>LHY</b>	0.28	<b>0.49</b>	<b>0.39</b>	0.32	0.32	0.28	<b>0.42</b>	-0.30	<b>0.48</b>	<b>0.45</b>	<b>0.46</b>	<b>-0.40</b>
<b>DEFSPR</b>	-0.18	-0.21	-0.26	-0.15	-0.25	-0.21	-0.26	0.18	-0.15	-0.17	-0.21	0.10
<b>FRBI</b>	0.01	0.19	0.06	-0.01	-0.12	-0.06	-0.05	0.10	-0.12	-0.01	-0.24	0.27
<b>GSCI</b>	-0.08	0.04	0.03	0.07	0.05	0.13	-0.05	0.03	0.18	0.12	0.19	-0.12

**Table 3: Correlation between the Hedge Fund Indexes and Risk Factors during different market conditions**

This table shows the results of the following regressions for eight HFR and four CSFB/Tremont hedge fund indexes during January 1990 to June 2000 for HFR and January 1994 to June 2000 for CSFB/Tremont:

$$R_t^i = a_0^i + b_0^i RUS_t + b_1^i D + g^i D \times RUS_t + e_t^i$$

where  $R_t^i$  are the returns on hedge fund index  $i$  during month  $t$ ,  $a_0^i$  are the intercepts for hedge fund index  $i$ ,  $b_0^i$  are the slope coefficients on Russell 3000 index,  $b_1^i$  are the slope coefficients on the dummy variable  $D$  ( $D = 1$  if return for Russell 3000 index is less than its median return and  $D = 0$  if return for Russell 3000 index is equal to or more than the median return),  $g^i$  are the slope coefficients on the interaction terms  $D \times RUS_t$  and  $e_t^i$  are the error terms. Various hedge fund strategies are Event Arbitrage (EA), Restructuring (REST), Event Driven (ED), Relative Value Arbitrage (RVAL), Convertible Arbitrage (CA), Equity Hedge or Long/Short Equity (EH), Equity Non-Hedge (ENH) and Short Selling or Dedicated Short-Bias (SHORT). Parameters significantly different from zero at the 5% level are shown in bold face.

	HFR								CFSB/TREMONT			
	EA	REST	ED	RVAL	CA	EH	ENH	SHORT	ED	CA	EH	SHORT
$a_0$	<b>1.24</b>	<b>2.51</b>	<b>2.40</b>	<b>1.88</b>	<b>1.22</b>	<b>2.27</b>	<b>1.94</b>	0.50	<b>1.88</b>	<b>1.89</b>	1.92	0.12
$b_0$	0.03	-0.18	-0.04	-0.10	0.00	0.21	<b>0.51</b>	<b>-0.90</b>	-0.02	-0.18	0.31	<b>-0.76</b>
$b_1$	0.01	-0.88	<b>-1.08</b>	-0.63	-0.28	-0.98	-1.04	1.10	<b>-0.89</b>	-1.07	-1.45	1.05
$g$	<b>0.31</b>	<b>0.67</b>	<b>0.58</b>	<b>0.35</b>	<b>0.16</b>	<b>0.32</b>	<b>0.52</b>	-0.34	<b>0.55</b>	0.31	0.42	-0.46
Adj. R <sup>2</sup>	33.45	34.93	55.15	26.53	17.17	45.33	67.16	49.75	51.48	4.10	48.84	65.89

**Table 4: Results with HFR Equally-Weighted Indexes**

This table shows the results of the regression  $R_t^i = c^i + \sum_{k=1}^K I_k^i F_{k,t} + u_t^i$  for the eight HFR indexes during the full sample period from January 1990 to June 2000 period. The table shows the intercept (C), statistically significant (at five percent level) slope coefficients on the various buy-and-hold and option-based risk factors and adjusted  $R^2$  (Adj- $R^2$ ). The buy-and-hold risk factors are Russell 3000 index (RUS), lagged Russell 3000 index (LRUS)), MSCI excluding the US index (MXUS), MSCI Emerging Markets index (MEM), Fama-French Size and Book-to-Market factors (SMB & HML), Momentum factor (MOM), Salomon Brothers Government and Corporate Bond index (SBG), Salomon Brothers World Government Bond index (SBW), Lehman High Yield Composite index (LHY), Federal Reserve Bank Competitiveness-Weighted Dollar index (FRBI), Goldman Sachs Commodity index (GSCI) and the change in the default spread in basis points (DEFSPR). The option-based risk factors include the at-the-money and out-of-the-money call and put options on the S&P 500 Composite index ( $SPC_{a/o}$  and  $SPP_{a/o}$ ). For the two call and put option-based strategies, subscripts  $a$  and  $o$  refer to at-the-money and out-of-the-money respectively.

Event Arbitrage		Restructuring		Event Driven		Relative Value Arbitrage		Convertible Arbitrage		Equity Hedge		Equity Non-Hedge		Short Selling	
Factors	$I$	Factors	$I$	Factors	$I$	Factors	$I$	Factors	$I$	Factors	$I$	Factors	$I$	Factors	$I$
C	0.04	C	0.43	C	0.20	C	0.38	C	0.24	C	0.99	C	0.56	C	-0.07
SPP <sub>o</sub>	-0.92	SPP <sub>o</sub>	-0.63	SPP <sub>o</sub>	-0.94	SPP <sub>o</sub>	-0.64	SPP <sub>a</sub>	-0.27	RUS	0.41	RUS	0.75	SPC <sub>o</sub>	-1.38
SMB	0.15	SMB	0.24	SMB	0.31	MOM	-0.08	LRUS	0.10	SMB	0.33	SMB	0.58	RUS	-0.69
HML	0.08	HML	0.12	HML	0.12	SMB	0.17	SMB	0.05	HML	-0.08	MEM	0.05	SMB	-0.77
		LRUS	0.06	RUS	0.17	HML	0.08	MEM	0.03	GSCI	0.08			HML	0.40
		LHY	0.13	MEM	0.06	MXUS	0.04	SBG	0.16						
		FRBI	0.27												
		MEM	0.09												
Adj- $R^2$	44.04	Adj- $R^2$	65.57	Adj- $R^2$	73.38	Adj- $R^2$	52.17	Adj- $R^2$	40.51	Adj- $R^2$	72.53	Adj- $R^2$	91.63	Adj- $R^2$	82.02

**Table 5: Results with CSFB/Tremont Value-Weighted Indexes**

This table shows the results of the regression  $R_t^i = c^i + \sum_{k=1}^K I_k^i F_{k,t} + u_t^i$  for the four CSFB/Tremont indexes during the full sample period from January 1994 to June 2000. The table shows the intercept (C), statistically significant (at five percent level) slope coefficients on the various buy-and-hold and option-based risk factors and adjusted  $R^2$  (Adj- $R^2$ ). The buy-and-hold risk factors are Russell 3000 index (RUS), lagged Russell 3000 index (LRUS), MSCI excluding the US index (MXUS), MSCI Emerging Markets index (MEM), Fama-French Size and Book-to-Market factors (SMB & HML), Momentum factor (MOM), Salomon Brothers Government and Corporate Bond index (SBG), Salomon Brothers World Government Bond index (SBW), Lehman High Yield Composite index (LHY), Federal Reserve Bank Competitiveness-Weighted Dollar index (FRBI), Goldman Sachs Commodity index (GSCI) and the change in the default spread in basis points (DEFSPR). The option-based risk factors include the at-the-money and out-of-the-money call and put options on the S&P 500 Composite index ( $SPC_{a/o}$  and  $SPP_{a/o}$ ). For the two call and put option-based strategies, subscripts  $a$  and  $o$  refer to at-the-money and out-of-the-money respectively.

<b>Event Driven</b>		<b>Convertible Arbitrage</b>		<b>Long/Short Equity</b>		<b>Short Selling</b>	
<b>Factors</b>	<b><i>I</i></b>	<b>Factors</b>	<b><i>I</i></b>	<b>Factors</b>	<b><i>I</i></b>	<b>Factors</b>	<b><i>I</i></b>
C	0.59	C	0.59	C	0.26	C	0.40
SPP <sub>o</sub>	-0.66	LRUS	0.09	HML	-0.25	RUS	-1.03
SMB	0.08	SBW	-0.20	RUS	0.53	SMB	-0.42
MEM	0.08	LHY	0.41	SMB	0.31	DEFSPR	-0.32
LHY	0.50					MOM	0.22
SBG	-0.94					HML	0.19
DEFSPR	-0.46						
Adj- $R^2$	73.55	Adj- $R^2$	33.35	Adj- $R^2$	83.50	Adj- $R^2$	84.97

**Table 6: T-test and Wilcoxon sign test results for difference in mean and median returns of HFR and CSFB/Tremont Hedge Fund Indexes and their Replicating Portfolios during the out-of-sample period (July 2000 to Dec 2001)**

This table shows the results of two-sided heteroskedastic t-test and Wilcoxon sign test for difference in the mean and median returns of eight HFR and four CSFB/Tremont indexes and those of their corresponding replicating portfolios using our model (i.e. using both buy-and-hold and option-based risk factors) during the out-of-sample period from July 2000 to Dec 2001.  $\Delta r$  is mean (median) return of the index minus that of its replicating portfolio for the t-test and Wilcoxon sign test respectively. # indicates  $\Delta r$  is significantly different from zero at 5% level.

Hedge Fund Strategy		HFR		CSFB/Tremont	
		t-test	Sign test	t-test	Sign test
Event Arbitrage	$\Delta r$	-0.082	0.050		
	p-value	0.935	1.000		
Restructuring	$\Delta r$	-0.215	0.023		
	p-value	0.831	0.815		
Event Driven	$\Delta r$	0.246	0.840	1.216	1.010
	p-value	0.808	1.000	0.238	0.096
Relative Value Arbitrage	$\Delta r$	-0.066	0.494		
	p-value	0.948	1.000		
Convertible Arbitrage	$\Delta r$	1.988	0.516	2.265 <sup>#</sup>	1.132 <sup>#</sup>
	p-value	0.115	0.238	0.033	0.031
Equity Hedge (Long/Short Equity)	$\Delta r$	0.186	-0.161	0.450	0.377
	p-value	0.854	0.481	0.657	0.481
Equity Non-Hedge	$\Delta r$	-0.220	-0.516		
	p-value	0.827	0.815		
Short Selling (Dedicated Short-Bias)	$\Delta r$	0.035	-0.469	-0.168	-1.918
	p-value	0.973	0.815	0.868	0.815

**Table 7: Out-of-sample Regression results with Individual Hedge Funds using Replicating Portfolios**

The following table shows the distribution of the adjusted- $R^2$  (in terms of the number of funds falling in different ranges of  $R^2$  values, mean and median  $R^2$  values) from the following out-of-the-sample regressions:

$$R_{j,t}^i = a^i + b^i RP_{j,t} + e_t^i$$

where

$R_{j,t}^i$  = net-of-fees excess return (in excess of the risk-free rate of interest) on an individual hedge fund  $i$  belonging to hedge fund strategy  $j$  during month  $t$ , and  $RP_{j,t}$  = excess return on the replicating portfolio to strategy  $j$  during month  $t$ . We consider individual hedge funds following eight different strategies (Event Arbitrage (EA), Restructuring (REST), Event Driven (ED), Relative Value Arbitrage (RVAL), Convertible Arbitrage (CA), Equity Hedge (EH), Equity Non-Hedge (ENH) and Short Selling (SS)) from HFR database on the excess returns of the HFR hedge fund index replicating portfolios during July 2000-Aug 2001 period and individual hedge funds following four different strategies (Event Driven (ED), Convertible Arbitrage (CA), Long/Short Equity (L-S E) and Dedicated Short-Bias (DSB)) from TASS+ database on the CSFB/Tremont hedge fund index replicating portfolios during Jul 2000-Aug 2001 period.

Range of $R^2$	HFR								TASS+			
	Number of funds								Number of funds			
	EA	REST	ED	RVAL	CA	EH	ENH	SS	ED	CA	L-S E	DSB
Less than -20%	0	0	1	0	0	0	0	0	0	0	1	0
-20 - -10%	5	0	3	0	1	8	1	0	7	5	30	0
-10 – 0%	7	6	48	7	29	99	9	0	12	5	34	1
0 - 10%	5	3	14	0	10	28	3	0	11	2	30	0
10 - 20%	1	4	11	2	8	29	4	1	9	6	23	0
20 - 30%	3	3	8	0	5	39	4	1	5	2	17	0
30 - 40%	0	1	7	1	0	37	7	1	9	0	21	0
40 - 50%	0	1	13	1	2	35	2	0	6	3	27	0
50 - 60%	0	0	12	0	2	40	7	2	6	2	17	0
60 - 70%	0	0	2	1	1	44	7	4	4	3	13	1
70 - 80%	0	1	2	0	0	35	7	4	3	3	20	1
80 - 90%	0	0	1	0	0	21	5	2	2	1	14	1
90 - 100%	0	0	0	0	0	1	2	0	0	0	2	2
Mean	0.3	13.9	15.4	9.6	6.0	32.1	41.7	60.9	23.8	24.8	27.5	67.9
Median	-5.0	11.0	5.6	-5.8	-2.8	31.8	43.1	61.0	19.2	18.1	24.1	81.1

**Table 8: Out-of-sample Regression results with Individual Hedge Funds using Indexes**

The following table shows the distribution of the adjusted-R<sup>2</sup> (in terms of the number of funds falling in different ranges of R<sup>2</sup> values, mean and median R<sup>2</sup> values) from the following out-of-the-sample regressions:

$$R_{j,t}^i = \mathbf{a}^i + \mathbf{b}^i I_{j,t} + e_t^i$$

where

$R_{j,t}^i$  = net-of-fees excess return (in excess of the risk-free rate of interest) on an individual hedge fund  $i$  belonging to hedge fund strategy  $j$  during month  $t$ , and  $I_{j,t}$  = excess return on the index for strategy  $j$  during month  $t$ . We consider individual hedge funds following eight different strategies (Event Driven (ED), Relative Value Arbitrage (RVA), Equity Hedge (EH), Equity Non-Hedge (ENH), Short Selling (SS), Event Arbitrage (EA) and Restructuring (REST)) from HFR database on the excess returns of the HFR hedge fund index replicating portfolios during Jul00-Aug01 period and individual hedge funds following four different strategies (Event Driven (ED), Convertible Arbitrage (CA), Long/Short Equity (L-S E) and Dedicated Short-Bias (DSB)) from TASS+ database on the CSFB/Tremont hedge fund index replicating portfolios during Jul00-Aug01 period.

Range of R <sup>2</sup>	HFR								TASS+			
	Number of funds								Number of funds			
	EA	REST	ED	RVAL	CA	EH	ENH	SS	ED	CA	L-S E	DSB
Less than -20%	0	0	0	0	0	0	0	0	0	0	1	0
-20 - -10%	0	0	4	0	1	9	1	0	10	3	43	0
-10 – 0%	2	5	39	4	7	77	10	0	11	5	48	0
0 - 10%	3	2	18	2	6	52	1	0	12	3	31	1
10 - 20%	6	3	11	2	10	30	5	1	4	3	20	0
20 - 30%	1	3	12	1	4	36	4	0	6	2	17	0
30 - 40%	3	4	7	1	5	23	4	1	9	5	14	1
40 - 50%	1	0	15	1	3	47	7	0	6	1	14	0
50 - 60%	1	1	6	0	6	45	5	3	8	1	14	0
60 - 70%	1	1	8	0	9	43	5	3	6	3	19	0
70 - 80%	0	0	1	1	4	41	6	1	0	5	17	2
80 - 90%	2	0	1	0	3	12	9	3	1	1	10	1
90 - 100%	1	0	0	0	0	1	1	3	1	0	1	1
Mean	31.0	18.9	17.9	16.1	35.0	32.2	41.8	68.6	22.1	31.1	21.4	59.8
Median	18.9	17.8	9.5	8.4	30.9	32.3	45.1	66.4	17.7	28.9	11.2	75.4



**Table 9: Conditional Value-at-Risk for Mean-Variance and Mean-Conditional Value-at-Risk Efficient Portfolios**

This table shows the conditional Value-at-Risk (CVaR) figures (reported as the magnitude of losses) at different confidence levels for Mean Variance (M-V) and Mean-CVaR (M-CVaR) efficient portfolios constructed using monthly returns of the eight HFR hedge fund strategies from January 1990 to June 2000.  $s$  indicates the volatility of portfolio returns and Ratio is the ratio of CVaR of M-V efficient portfolio to that of M-CVaR efficient portfolio for the same portfolio volatility.  $\Delta m$  is the difference in the mean returns (in basis points) of the M-CVaR and M-V efficient portfolios for the same portfolio volatility,  $s$ .

s	90%				95%				99%			
	CVaR (M-V)	CVaR (M-CVaR)	Ratio	$\Delta m$	CVaR (M-V)	CVaR (M-CVaR)	Ratio	$\Delta m$	CVaR (M-V)	CVaR (M-CVaR)	Ratio	$\Delta m$
0.73	0.42	0.33	1.25	-7.04	0.88	0.51	1.75	-7.24	2.41	0.88	2.73	-17.15
0.74	0.35	0.29	1.23	-5.51	0.85	0.51	1.67	-5.73	2.52	0.97	2.60	-14.73
0.76	0.33	0.27	1.22	-4.10	0.82	0.53	1.55	-4.30	2.63	1.07	2.45	-12.47
0.80	0.33	0.27	1.21	-2.65	0.81	0.55	1.48	-2.90	2.74	1.29	2.12	-10.85
0.85	0.33	0.28	1.17	-1.30	0.84	0.60	1.40	-1.55	2.84	1.49	1.91	-9.26
0.92	0.36	0.31	1.15	-1.16	0.90	0.69	1.30	-1.36	2.88	1.70	1.70	-7.72
1.00	0.43	0.38	1.14	-1.04	0.97	0.77	1.25	-1.14	2.86	1.90	1.50	-6.40
1.10	0.51	0.45	1.12	-0.93	1.05	0.88	1.20	-0.96	2.83	2.10	1.35	-5.10
1.21	0.61	0.55	1.11	-0.81	1.20	1.04	1.15	-0.85	2.83	2.36	1.20	-4.90
1.33	0.75	0.68	1.10	-0.68	1.39	1.25	1.11	-0.72	2.86	2.51	1.14	-2.30
1.47	0.91	0.84	1.08	-0.54	1.59	1.48	1.08	-0.58	3.18	2.94	1.08	-1.34
1.61	1.11	1.05	1.06	-0.40	1.87	1.76	1.06	-0.44	3.54	3.31	1.07	-1.01
1.78	1.38	1.31	1.05	-0.26	2.16	2.04	1.06	-0.30	4.31	4.07	1.06	-0.74
1.97	1.68	1.59	1.05	-0.14	2.49	2.35	1.06	-0.21	5.09	4.85	1.05	-0.51
2.17	1.98	1.89	1.05	-0.08	2.88	2.73	1.06	-0.12	5.86	5.58	1.05	-0.23
2.37	2.30	2.20	1.04	-0.03	3.28	3.12	1.05	-0.08	6.64	6.38	1.04	-0.12
2.59	2.61	2.51	1.04	-0.01	3.67	3.51	1.05	-0.04	7.41	7.13	1.04	-0.07
<b>AVG.</b>			<b>1.12</b>				<b>1.25</b>				<b>1.54</b>	

**Table 10: Summary Statistics of Systematic Returns of HFR Hedge Fund Indexes**

This table shows the mean returns, standard deviations (SD), medians, minimum realizations, maximum realizations and Conditional Value-at-Risk (reported as the magnitude of losses) at 90%, 95% and 99% confidence levels for the systematic returns of eight HFR hedge fund indexes during our sample period from January 1990 to June 2000 (Panel A) and before our sample period from January 1927 to December 1989 (Panel B).

**Panel A: Recent Returns – January 1990 to June 2000**

Hedge fund strategy	Mean	SD	Median	Min.	Max.	CVaR (90%)	CVaR (95%)	CVaR (99%)
<i><u>Non-Directional</u></i>								
Event Arbitrage	1.00	0.88	1.18	-3.31	2.40	1.00	1.86	3.31
Restructuring	1.26	1.49	1.53	-5.30	4.88	1.83	3.10	5.30
Event Driven	1.08	1.61	1.50	-6.66	4.40	2.25	3.54	6.66
Relative Value Arbitrage	0.82	0.89	0.94	-3.22	3.03	0.91	1.62	3.22
Convertible Arbitrage	0.83	0.65	0.91	-1.90	1.99	0.46	0.95	1.90
Equity Hedge	0.81	2.24	0.89	-8.54	7.82	3.16	4.46	8.54
<i><u>Directional</u></i>								
Equity Non-Hedge	1.17	3.90	1.61	-16.11	10.08	6.22	8.37	16.11
Short Selling	0.15	5.81	0.10	-18.54	20.95	9.95	12.78	18.54

**Panel B: Long-run returns – January 1927 to December 1989**

Hedge fund strategy	Mean	SD	Median	Min.	Max.	CVaR (90%)	CVaR (95%)	CVaR (99%)
<i><u>Non-Directional</u></i>								
Event Arbitrage	0.72	1.45	0.95	-7.76	7.81	2.45	3.47	5.71
Restructuring	0.97	2.40	1.25	-11.11	18.78	3.99	5.56	8.53
Event Driven	0.85	2.64	1.16	-11.73	19.94	4.38	5.96	9.18
Relative Value Arbitrage	0.61	1.46	0.70	-6.37	10.16	2.23	3.12	5.12
Convertible Arbitrage	0.57	0.97	0.66	-3.97	6.57	1.41	1.97	3.05
Equity Hedge	0.60	2.69	0.66	-11.70	19.32	4.26	5.71	9.30
<i><u>Directional</u></i>								
Equity Non-Hedge	0.96	5.53	1.20	-23.43	39.87	8.95	11.77	18.82
Short Selling	0.00	6.27	0.05	-39.72	26.94	11.08	14.76	25.94

**Table 11: Summary Statistics of Systematic Returns of CSFB/Tremont Hedge Fund Indexes**

This table shows the mean returns, standard deviations (SD), medians, minimum realizations, maximum realizations and Conditional Value-at-Risk (reported as the magnitude of losses) at 90%, 95% and 99% confidence levels for the systematic returns of four CSFB/Tremont hedge fund indexes during the sample period (January 1994 to June 2000) (Panel A) and before the sample period from January 1927 to December 1993 (Panel B).

**Panel A: Recent Returns – January 1994 to June 2000**

Hedge fund strategy	Mean	SD	Median	Min.	Max.	CVaR (90%)	CVaR (95%)	CVaR (99%)
<i><b>Non-Directional</b></i>								
<b>Event Driven</b>	1.26	1.56	1.56	-6.29	4.16	1.85	2.98	6.29
<b>Convertible Arbitrage</b>	0.91	0.68	1.04	-1.57	1.84	0.49	0.97	1.57
<b>Long/Short Equity</b>	1.16	3.38	1.01	-11.61	10.86	5.05	7.00	11.61
<i><b>Directional</b></i>								
<b>Dedicated Short-Bias</b>	-0.55	4.88	-0.83	-9.73	21.60	7.28	8.26	9.73

**Panel B: Long-run returns – January 1927 to December 1993**

Hedge fund strategy	Mean	SD	Median	Min.	Max.	CVaR (90%)	CVaR (95%)	CVaR (99%)
<i><b>Non-Directional</b></i>								
<b>Event Driven</b>	0.83	2.27	1.17	-10.83	15.60	3.92	5.44	8.65
<b>Convertible Arbitrage</b>	0.59	1.00	0.70	-4.31	5.50	1.45	2.13	3.36
<b>Long/Short Equity</b>	0.62	3.23	0.77	-15.35	18.52	5.38	7.12	12.02
<i><b>Directional</b></i>								
<b>Dedicated Short-Bias</b>	-0.18	6.65	-0.41	-55.01	29.85	11.64	16.08	33.86

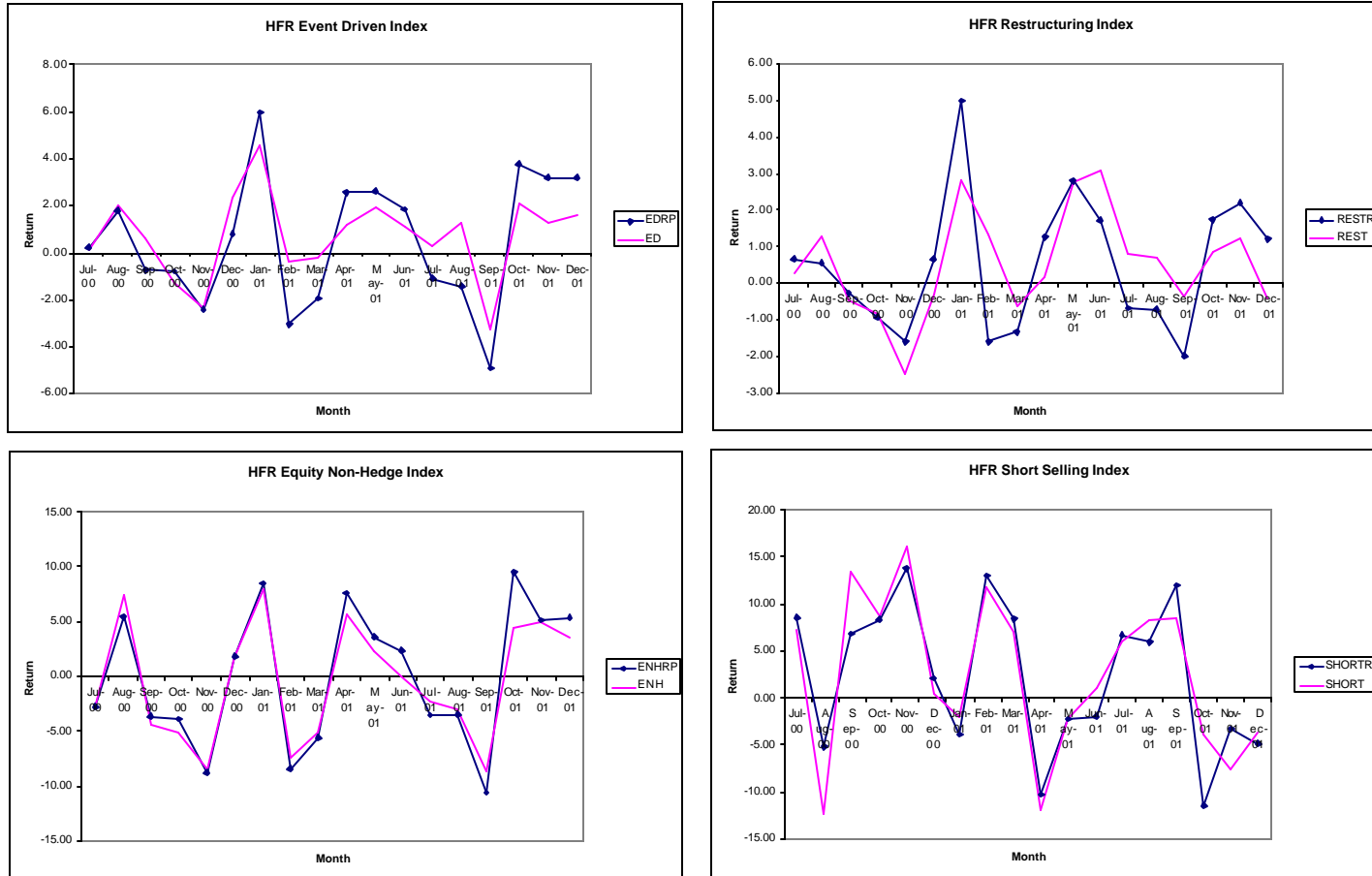
**Table 12: T-test, Wilcoxon sign test and Variance Ratio test results for difference in mean, median and standard deviation of systematic returns of HFR and CSFB/Tremont Hedge Fund Indexes**

This table shows the results of two-sided heteroskedastic t-test, Wilcoxon sign test and Variance Ratio (VR) test for difference in the mean, median and standard deviation of systematic returns of eight HFR and four CSFB/Tremont indexes during the pre-sample period (Jan 27 to Dec 89 for HFR and Jan 27 to Dec 93 for CSFB/Tremont) and those during the sample period (Jan 90 to Jun 00 for HFR and Jan 94 to Jun 00 for CSFB/Tremont).  $\Delta$  is the difference in the mean (t-test), median (sign test) and standard deviation (VR test) of the systematic returns during the pre-sample and sample period. # indicates that the difference  $\Delta$  is significantly different from zero at 10% level.

Hedge Fund Strategy		HFR			CSFB/Tremont		
		t-test	Sign test	VR test	t-test	Sign test	VR test
Event Arbitrage	$\Delta$	-0.278 <sup>#</sup>	-0.231 <sup>#</sup>	0.559 <sup>#</sup>			
	p-value	0.004	0.000	0.000			
Restructuring	$\Delta$	-0.288	-0.274	0.904 <sup>#</sup>			
	p-value	0.071	0.247	0.000			
Event Driven	$\Delta$	-0.229	-0.342	1.029 <sup>#</sup>	-0.433 <sup>#</sup>	-0.397	0.714 <sup>#</sup>
	p-value	0.186	0.247	0.000	0.028	0.428	0.000
Relative Value Arbitrage	$\Delta$	-0.211 <sup>#</sup>	-0.243 <sup>#</sup>	0.578 <sup>#</sup>			
	p-value	0.026	0.002	0.000			
Convertible Arbitrage	$\Delta$	-0.252 <sup>#</sup>	-0.242 <sup>#</sup>	0.322 <sup>#</sup>	-0.318 <sup>#</sup>	-0.339 <sup>#</sup>	0.321 <sup>#</sup>
	p-value	0.000	0.001	0.000	0.000	0.004	0.000
Equity Hedge (Long/Short Equity)	$\Delta$	-0.210	-0.237	0.448 <sup>#</sup>	-0.538	-0.235	-0.152
	p-value	0.347	0.789	0.008	0.162	0.428	0.588
Equity Non-Hedge	$\Delta$	-0.207	-0.409	1.636 <sup>#</sup>			
	p-value	0.607	0.789	0.000			
Short Selling (Dedicated Short-Bias)	$\Delta$	-0.146	-0.043	0.457	0.374	0.421	1.779 <sup>#</sup>
	p-value	0.808	0.247	0.269	0.534	0.428	0.000

**Figure 1: Out-of-Sample Results for HFR strategies**

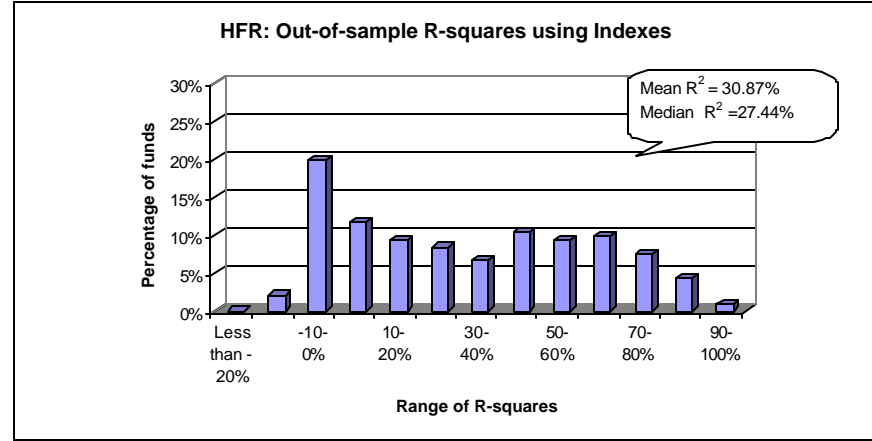
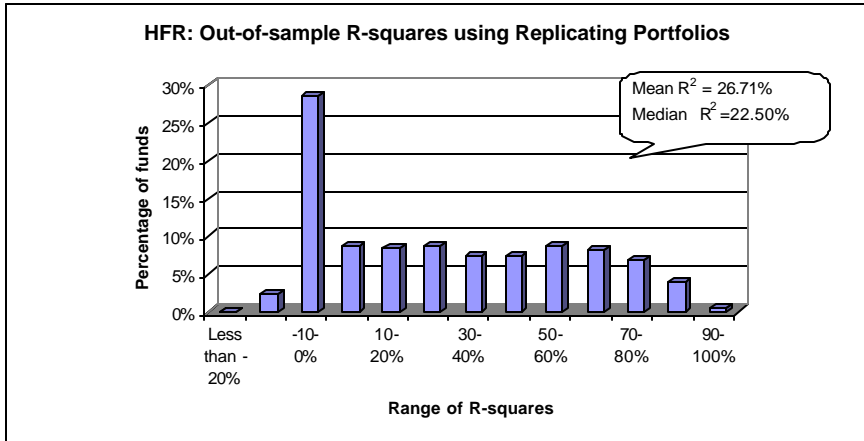
This figure plots the returns for the replicating portfolios and the actual HFR index returns during the out-of-sample period from July 2000 to December 2001. EDRP, RESTRP, HLBRP and SHORTRP are the replicating portfolios for HFR’s Event Driven (ED), Restructuring (REST), Equity Non-Hedge (ENH) and Short Selling (SHORT) hedge fund strategies constructed using buy-and-hold and option-based risk factors estimated during our sample period from January 1990 to June 2000.



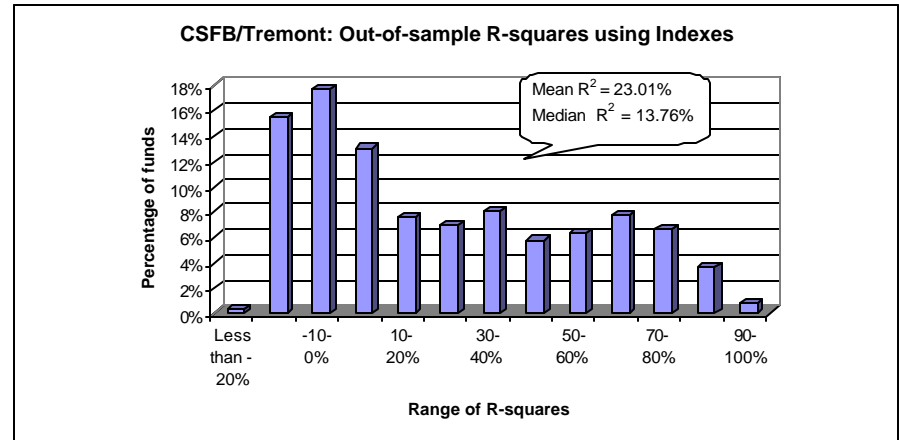
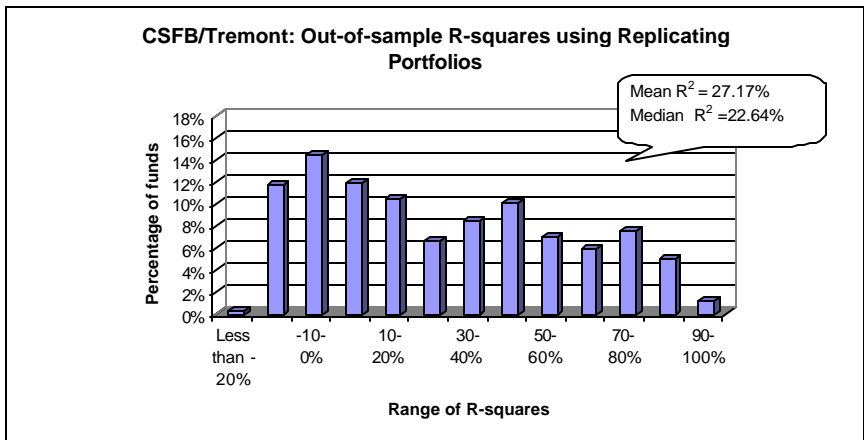
**Figure 2: Distribution of Out-of-Sample R-Squares for Individual HFR and CSFB/Tremont Hedge Funds**

The following figures show the distribution of out-of-sample Rsquares from regressions of the excess returns on individual hedge funds in HFR and CSFB/Tremont databases on the excess returns of their corresponding index replicating portfolios and on the excess returns of their corresponding indexes.

**Panel A: Results for individual HFR hedge funds**



**Panel B: Results for individual CSFB/Tremont hedge funds**



**Figure 3: Ratio of Conditional Value-at-Risk for Mean-Variance and Mean-Conditional Value-at-Risk Efficient Portfolios**

This figure plots the ratio of the Conditional Value-at-Risk (CVaR) for Mean-Variance and Mean-CVaR efficient portfolios at different confidence levels for different levels of portfolio volatility. The efficient portfolios are constructed using monthly returns of eight HFR hedge fund strategies during our sample period from January 1990 to June 2000.

