Benefits and Risks of Alternative Investment Strategies*

Noël Amenc
Professor of Finance at Edhec
Director of Research and Development, Misys Asset Management Systems

Lionel Martellini
Assistant Professor of Finance at the Marshall School of Business, USC Los Angeles

Mathieu Vaissié
Research Engineer with Edhec

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Introduction

The growth of alternative investment has been considerable in recent years. For both institutional and private investors, it seems that alternative investment now constitutes a distinct class within their overall asset allocation.

A recent survey of institutional investors carried out by Goldman Sachs and Frank Russell[1] revealed that the respondents invested more than 1.7% of their assets in hedge funds in 2001 and plan to increase the investment to 3.4% for 2003. More generally, the year 2001 represented a record year for investments in hedge funds, bringing together more than 30 billion dollars.

In Europe, the alternative investment industry experienced a growth rate of 60% in 2000 and 40% in 2001 and the major asset management firms and institutional investors are forecasting a long-term (5-year) growth rate of more than 20%.

In a relatively difficult context for the asset management industry, the alternative class (cf. Appendix 1 for a short description of the main alternative strategies) represents a commercial eldorado. This attractiveness is reinforced by the difficult stock market situation, which increases investors' interest in investment services that base their strategy on the decorrelation with the risks and returns of the financial markets and therefore the search for an absolute return.

Consequently, hedge funds, which are often referred to as "pure alpha" funds, warrant significant remuneration, determined not only on the basis of managed assets, but also on outperformance compared to the risk-free rate. That form of remuneration and commercial arguments that equate hedge funds to low-risk investments, because they have both low correlation with the risks of the financial markets and low volatility, obviously attracts the attention of investors and the regulatory authorities.

Apart from evaluating the operational risks that may be incurred by funds that are managed in unregulated zones and that invest in instruments traded on markets that are themselves unregulated, it is appropriate to enquire into the nature of the financial risks of alternative investments.

Can we be satisfied, as we have observed from the arguments of European asset management firms, with evaluating the risk-adjusted return of hedge funds through the Sharpe ratio alone?

The aim of our article is to lay the foundations for a reasoned discussion of the subject. What are the risks involved in alternative investment and how can alternative investment be used to diversify investors' portfolios?

In addition, to analyse the challenges and opportunities that hedge funds represent in terms of risk management for investors, we initially present the arguments in favour of alternative diversification. In the second part of the article, we will discuss the need for an appropriate risk measure, along with the specific difficulties related to the monitoring of risks.

[1] Goldman Sachs and Frank Russell, “Alternative investing by tax exempt organizations”, 2001.

I. The advantages of alternative diversification

Investors' interest in hedge funds can be explained in particular through the fact that alternative funds actually present real diversification strengths through their exposure to risks other than market risks.

These strengths are all the more attractive in a context of relative decline of investment opportunities in traditional asset classes due to the low degree of diversification offered by a purely geographical or sectorial asset distribution. It is actually well known that the limitations of international diversification tend to take effect at the exact moment that the investor has a need for it, namely in periods when the markets drop significantly (see for example Longin and Solnik (1995)). In short, the correlation between the stock markets in different countries converges towards 1 when there is a sharp drop in the American markets.

Conversely, it seems that the diversification offered by hedge funds, or to be more precise, certain hedge funds, is relatively stable: the conditional correlations (calculated from a sample that only contains periods corresponding to the most significant decreases or increases in a traditional reference index) between the returns on alternative funds and those of stock and bond market indices are relatively similar to the unconditional correlations (on this point, see also Schneeweis and Spurgin (1999)). For example, the Market Neutral and Macro strategies retain a stable market risk exposure whatever the market conditions.

Conditional correlations of hedge fund styles (HFR) with the stock market (S&P 500)
(02/1990 – 10/2001)

Correlation coefficients	Market falling significantly (1)	Market stable (2)	Market rising significantly (3)	(1) - (3)	Type of correlation
Convertible Arbitrage	0,49	0,30	0,07	***0,42***	**Unfavourable***
Distressed Securities	0,64	0,32	-0,18	***0,82***	
Emerging Markets	0,75	0,32	0,32	***0,43***	
Equity Hedge	0,60	0,46	0,15	***0,45***	
Relative Value	0,58	0,24	-0,28	***0,86***	
Equity Non-Hedge	0,74	0,59	0,36	***0,38***	
Event Driven	0,79	0,54	-0,12	***0,91***	
Fixed Income Arbitrage	0,68	0,47	-0,13	***0,82***	
Market Neutral	0,03	0,05	-0,06	***0,09***	**Stable****
Macro	0,29	0,18	0,21	***0,08***	
Short Selling	-0,48	-0,59	-0,34	***-0,14***	**Favourable*****
Market Timing	0,15	0,53	0,29	***-0,14***	

** (1) - (3) > 0,10   ** - 0,10 < (1) - (3) < 0,10   *** (1) - (3) < - 0,10*

Moreover, for certain strategies (Convertible Arbitrage, Emerging Markets, Distressed Securities, Relative Value and Event Driven), we observe an increase in the correlation coefficient when market conditions deteriorate. This development is obviously not favourable for investors, since their diversification strategy will lose its effectiveness at precisely the moment when they need it the most. Other strategies, conversely, will see their correlation coefficient increase as the market performance improves. The investor will therefore be exposed to a rise in the market and hedged against a fall in the market! These particularly favourable strategies for the investor are the following: Market Timing and Short Selling.

It is nevertheless appropriate, at this stage, to recall the limitations of evaluating conditional correlations. More often than not, they are calculated on the basis of hedge fund performance compared to that of a sample of the market indices' best or worst months or days for a given period. The conditional correlations thereby obtained correspond not to extreme values, but to a mean that is itself sensitive to the sample chosen.

The integration of the risk of an increase in the correlation coefficients in extreme market conditions has been the subject of recent research, notably after the crisis in the summer of 1998. On this topic, Anson (2000) notes that the returns on composite indices of hedge fund styles, whether equally weighted (HFR) or proportional to the assets managed (CFSB/Tremont), were significantly affected by the debt market crisis (August 1998) and not by the near-bankruptcy of LTCM (September and October 1998). Therefore, the sudden and dramatic increase in the correlation coefficients, and not the systemic crisis in the alternative universe, was the main source of risk during that period.

The question of evaluating conditional correlations is actually one of the consequences of the non-linearity of alternative returns. We shall come back to these consequences in the second part of the article.

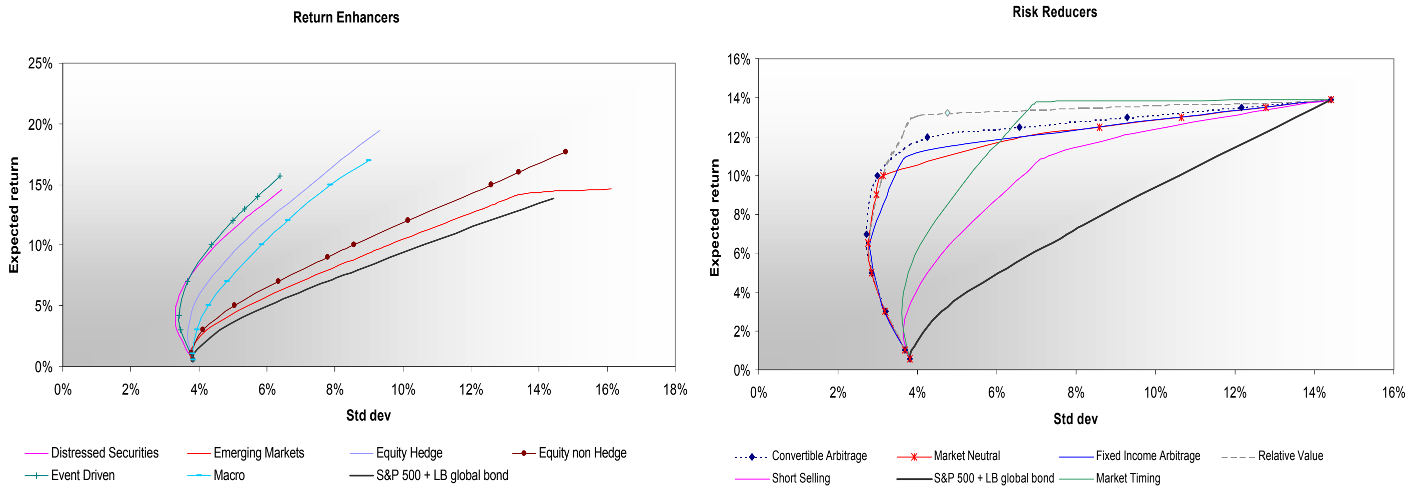
A detailed analysis of the correlation of hedge fund returns with those of traditional markets tends to prove that it is simplistic to consider those funds as being part of a homogenous asset class. There are actually a large number of alternative strategies, with each having different diversification capabilities. Certain strategies such as Market Neutral, Relative Value, or Convertible Arbitrage generally have a low level of correlation with the performances of the S&P 500, an American stock market index (correlations typically less than 0.5 as an absolute value), and with those of the Lehman Brothers US Aggregate Index, the reference bond index. However, for other strategies such as Equity Non Hedge or Short Selling, this is absolutely not the case.

Correlation with stock (S&P 500) and bond (LBGBI) indices (02/1990 – 10/2001)

Correlation Coefficients	**S&P 500**	**Lehman US**
Convertible Arbitrage	0.31	0.18
Distressed Securities	0.37	0.01
Emerging Markets	0.57	0.03
Equity Hedge	0.63	0.12
Market Neutral	0.12	0.23
Equity non-Hedge	0.77	0.13
Event Driven	0.59	0.10
Fixed Income Arbitrage	0.42	0.13
Macro	0.42	0.37
Relative Value	0.34	0.04
Short Selling	-0.69	-0.07
Market Timing	0.68	0.19

The variety of decorrelations compared to the returns on traditional assets allows us to envisage very diverse forms of alternative diversification, as is shown in the graphs below.

Diversification profiles: inclusion of hedge fund styles (HFR)
with stock (S&P 500) and bond (LBGBI) portfolios (02/1990 – 10/2001)



Following on from this, certain hedge funds have a high level of correlation with the market, and offer returns that are particularly high. Adding this type of fund to a portfolio made up of stocks and bonds would result in an increase in the expected return while retaining a high degree of volatility. Distressed Securities, Emerging Markets, Event Driven, or Global Macro present these characteristics; these strategies can therefore be seen as "Return Enhancers." Conversely, integrating certain alternative strategies with low exposure to market risk, or indeed negative exposure, will result in a lowering of the portfolio's volatility. The Convertible Arbitrage, Fixed Income Arbitrage, Market Neutral or Short Selling (negative correlation) strategies correspond to this profile. These strategies can therefore be seen as Risk Reducers, or even as Pure Diversifiers (Short Selling). It is important to note that even a tiny variation in the volatility, and especially the return, of the assets used to construct efficient frontiers can have a major influence on the shape of that efficient frontier. For that reason, it is necessary to pay particular attention to the consequences of taking survivorship bias into account when evaluating the performance and volatility of alternative strategies. We shall come back to this point in more detail later on in this article.

These differences in correlation with the stock and bond markets can be explained through a difference in exposure to a certain number of risk factors that explain the returns of the alternative and traditional asset classes. Recent research on the analysis of alternative fund performance (Fung and Hsieh (1997), Schneeweis and Spurgin (1999), Amenc, Curtis and Martellini (2002)) has highlighted the fact that alternative funds are not only exposed to market risk (unforeseeable variations in the prices of basic assets, stocks, bonds, etc.), measured by the traditional "beta," but also, as a result of the very nature of the strategies implemented, to volatility risks (unforeseeable variations in the variability of the prices), default risks (unforeseeable variations in the propensity of certain counterparties to no longer be able to respect their commitments) and liquidity risks (unforeseeable variations in the capacity to move quantities of assets in a "reasonable" time scale at market prices).

In the table below we present the performance correlations of the different strategies under consideration, with the principal sources of risk (apart from market risk) that affect the returns of financial instruments. This study was carried out using the performances of the different HFR indices over the period February 1993/October 2001[2].

Table of correlations between the different Hedge Fund strategies (HFR) and risk factors (02/1993 – 10/2001)

Correlation coefficients	Volatility	Exchange rate	Raw materials	Liquidity	Default	3 month US Treasury Bill	Slope of the yield curve
Convertible Arbitrage	-0.33	0.14	0.04	-0.05	0.10	0.08	-0.17
Distressed Securities	-0.50	0.06	0.13	-0.12	-0.06	-0.16	0.25
Emerging Markets	-0.48	0.03	0.07	-0.01	0.07	-0.21	0.27
Equity Hedge	-0.43	-0.14	0.20	-0.01	-0.04	0.04	-0.02
Market Neutral	-0.02	-0.11	-0.12	0.00	-0.08	0.16	-0.15
Equity non-Hedge	-0.50	-0.17	0.17	-0.04	-0.04	-0.01	0.07
Event Driven	-0.57	-0.03	0.15	-0.03	0.00	-0.06	0.09
Fixed Income Arbitrage	-0.41	0.15	0.09	-0.07	-0.02	-0.09	0.21
Macro	-0.35	0.19	-0.03	0.09	-0.07	-0.16	0.16
Relative Value	-0.41	0.03	0.10	-0.13	-0.01	-0.12	0.12
Short Selling	0.37	0.19	-0.15	0.05	0.03	0.02	-0.05
Market Timing	-0.31	-0.15	0.05	0.13	0.07	-0.07	0.01
S&P 500	-0.42	-0.14	0.01	0.08	-0.07	0.10	-0.03
Lehman US Aggregate	-0.14	0.02	-0.01	0.19	-0.07	0.11	-0.18

The data used to characterise the different sources of risk is as follows:

The volatility risk is measured by the relative price variations of the VIX contract, the underlying of which is the implicit volatility of the S&P 100
The currency risk is measured by the evolution of the exchange rate of the US dollar compared to a basket of foreign currencies
The raw material risk is measured by the relative price variations of a barrel of crude oil
The liquidity risk is measured by the evolution of the volume of securities exchanged on the NYSE
The default risk is measured by the relative variations of the differential between the returns on bonds rated Baa and Aaa by Moody's
The slope of the yield curve is obtained by calculating the difference between the rate of return of a bond with a 30-year maturity and that of a 3-month Treasury bill.

Moreover, part of the strategies' return comes from this exposure to different risks. For instance, it has often been observed that a certain number of hedge funds pursuing a "fixed-income arbitrage" type strategy acted as liquidity providers on fixed-income security markets that were exposed to default risk, a role typically taken on by the trading desks of the major investment banks. It is in fact natural to seek to use the multiple facets of risk, and therefore of return. It increases the degree of liberty in investment decisions. Although the existence of alternative alphas is sometimes questioned, alternative betas often correspond to risk premiums that are traditionally arbitrated by the players present in the market and, as a result, correspond to market prices (the volatility, or notably the credit, market).

[2] It should be pointed out that monthly calculation of the correlations tends to smooth the results and therefore smoothes the impact of changes in the factor values within the months.

II. The difficulties in measuring alternative investment risks

Wanting to use hedge funds as a risk diversification tool presupposes that we have mastery over the issues at stake in the control of hedge fund risks. Even though recent research work has given us a better understanding of the subject, it is certainly the area in which the most progress remains to be made.

As stressed by, for example, Lo (2001), Amenc, Curtis and Martellini (2002) and Fung and Hsieh (2001), there are in fact at least three reasons for alternative funds posing specific problems for measuring and controlling risks[3]:

- difficulties in accounting for the dimensions of credit and liquidity risks
- difficulty in developing relevant benchmarks
- difficulties in accounting for the dynamic and non-linear aspects of alternative risk.

Before going into these difficulties in more detail, it is important to note that it is not because a fund is a hedge fund that the risk-free asset is necessarily a good benchmark. While nearly all hedge funds highlight a so-called "absolute return" policy, the risk-free rate is only a good benchmark if the following two conditions are respected:

- assumption 1: the fund has a market beta equal to zero
- assumption 2: the CAPM is an appropriate model for the alternative universe.

While the first assumption is respected for certain types of alternative funds, such as equity market neutral or fixed-income arbitrage, it is certainly not valid for the other categories (cf. table page 6).

The second assumption is even more debatable. In order to have a better comprehension of the issues at stake in the performance of alternative funds, it is important to understand first of all that the excess return of a risky portfolio compared to the risk-free rate, as it is measured for a given sample, can, in general, come from three distinct sources, as described in the following equation:

Excess return of the portfolio = normal return + abnormal return + statistical noise

The first term, the "normal" return, corresponds to the market's fair reward for the risks to which the portfolio is exposed. It is therefore a premium (or premiums) for the risk(s), which can be evaluated with the help of a single factor model like the CAPM or with the help of a more general, multifactor model, justified on the theoretical side through equilibrium (Merton, 1973) or arbitrage (Ross, 1976) reasoning, and implemented in the business world by a certain number of firms such as, for example, BIRR, Quantal, Aptimum and BARRA.

In the alternative universe, as we mentioned earlier, recent research has highlighted the inadequacy of the CAPM and underlined the exposure of alternative returns to factors such as volatility risk, default risk and liquidity risk.

It is therefore wrong to state that a non-directional alternative strategy, i.e. a strategy that is not exposed to market risk (for example, fixed-income arbitrage, convertible arbitrage or zero-beta strategies), should be rewarded at the risk-free rate. That is an overly restrictive application of a single factor model (the CAPM) which does not allow for a good description of the behaviour of alternative funds. Financial risks, particularly in the alternative world, are actually multiform, and consequently risk premiums are numerous and, more often than not, unstable. For example, a mean return of 10% for a non-directional fund like LTCM, when that of the risk-free rate is 4%, does not necessarily reflect an abnormal return of 6%. Within the 6%, there could be volatility, liquidity or default risk premiums (or any other risk factor, the existence of which may have escaped the wisdom of the modellers, that is rewarded by the market at equilibrium). In total, the abnormal return, the fund's alpha, could very well be positive, negative or nil!

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[3] Other important problems to be taken into account are the operational risks and survivorship bias (see, for example, Fung and Hsieh (2001) on the latter point).

The second term, the “abnormal” return, represents the fruit of the portfolio managers' expertise. Portfolio managers sometimes manage to obtain additional profits that are not strictly justified in terms of exposure to risks that are rewarded by the market. Indeed, obtaining abnormal returns is precisely what active managers in general, and hedge fund managers in particular, devote their efforts to.

II.1 Credit and liquidity risks

Among the risk factors that should be taken into account to evaluate both the reality of the return and also the alternative investment risk, particular attention should be paid to the liquidity and credit risks, for two essential reasons:

- unlike other risks, where there is general consensus as to how to measure them, accounting for credit and liquidity risks still constitutes both an operational and a theoretical challenge;
- the near-bankruptcy of the LTCM fund significantly highlighted the interdependence between the two risks, which makes it difficult to model them separately, as the multifactor approach suggests. The multifactor approach is the dominant doctrine in analysing portfolio risk and return.

In the area of credit risk, the financial literature is particularly rich in terms of both modelling and pricing[4]. This abundance of models is a relative reflection of the lack of maturity of the practices (cf. the most recent research carried out on this subject for drawing up the new Basel accord) faced with risks with returns that are not normally distributed[5] and with diversification that is only effective after taking a very large number of positions into account. Even though the conceptual foundations of these models are very similar and constitute an application/extension of the work of Merton (1974)[6], the implementation conditions and empirical tests of the models, because they are still very approximate[7], give results that are not very robust on account of the assumptions chosen.

Integrating the interdependence between credit risk and liquidity risk should notably lead to the modelling of the consequences of using leverage effects in arbitrage operations. But today, with the exception of highly academic research, such as applying mathematical network theory to the construction of systemic measures of credit and liquidity risk[8], professionals do not have robust and simple microeconomic results at their disposal in this area.

Faced with these modelling difficulties, certain authors have proposed measuring the exposure to liquidity risk by using the degree of auto-correlation of fund returns as a measure of the fund's liquidity risk. Lo (2001), relies on the statistics of Ljung and Box (1978) to do this.

Auto-correlation coefficient analysis concentrates on the problem posed by the consequences of liquidity or illiquidity on asset prices and position valuation. It is therefore only a partial view of the liquidity risks, but it does nevertheless correspond to an area that is of considerable concern to professionals. Asness, Krail and Liew (2000) highlighted the risks posed by the valuation of illiquid positions. In the same way, a recent study carried out by Capital Market Risk Advisors (2001) observed that substantial price and valuation differences (30 – 40%) were related to the choice of valuation methods and/or valuation models for the least liquid assets (high yield and distressed bonds, private securities, OTC options, structured notes and mortgage derivatives).

Managing the valuations of illiquid positions allows hedge fund results to be smoothed and very attractive risk-adjusted return indicators to be constructed. The lack of liquidity is, in a way, conceived by managers, no longer as a risk but as a selling point for seducing investors (Brook and Kat, 2001). Therefore, calculated and manipulated prices might be the best explanation for the quarterly persistence of funds, highlighted by Agarwal and Naik (2000).

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[4] For a review of the literature, one could consult Kao (2000) and Saunder (1999)

[5] We will examine this point in part II.4

[6] For a comparative review of the models and their application, it would be useful to refer to Crouhy, Galai and Mark (2000), Gordy (2000) and Basle (1999)

[7] A critical analysis of the empirical tests of these models was carried out by Bohn (1999a, 1999b)

[8] Notably Watts and Strogatz (1998) and Watts (1999)

II.2 Benchmarking in alternative investment

Since we have concluded that the risk-free rate is certainly not an appropriate benchmark for all types of hedge funds, it remains to be seen what constitutes a good benchmark. It seems that the alternative investment industry is currently switching from "absolute return" logic to "relative return" logic. The principle therefore consists of comparing the return of a given fund to that of a portfolio of funds following the same strategy (peer benchmarking), or that of a representative index (index benchmarking).

The difficulties related to the development of quality indices, which are already evident in the traditional universe, are exacerbated in the alternative investment world, in the areas of both representativeness and purity (i.e. homogeneity of the data). Firstly, it should be noted that the logic of representativeness through capitalisation can only be applied to the alternative universe with great difficulty, since information on the assets managed by hedge funds is generally not available in real time. That is why all hedge fund indices, with the exception of the CSFB/Tremont indices, operate in equal weighting mode today.

Secondly, the fact that there is no obligation to publish performance in the alternative world renders access to exhaustive databases very difficult. One of the most widely use alternative indices, the EACM 100 index, is calculated from the performances of 100 hedge funds extracted from a universe that currently contains over 6,000 funds!

The challenge of purity is also very difficult to handle in the alternative universe. In a world where the competitive advantage of managers is largely based on the sophistication and confidentiality of their "proprietary" management techniques, the index providers (with the notable exception of Zurich) are generally content to use the self-proclaimed styles of the managers. There is however good reason to believe that some managers, faced with fewer and fewer opportunities, occasionally deviate significantly from their ostensible management style (a phenomenon called "style drift").

As a result of the difficulty in accessing data, the set of funds taken into account for a given management style varies enormously from one provider to another. Consequently, the returns on competing indices for the same segment of the alternative investment market can vary considerably.

The following table gives details on the maximal return differences for the period 1998-2000 between the monthly returns of the various competing indices for a given style.

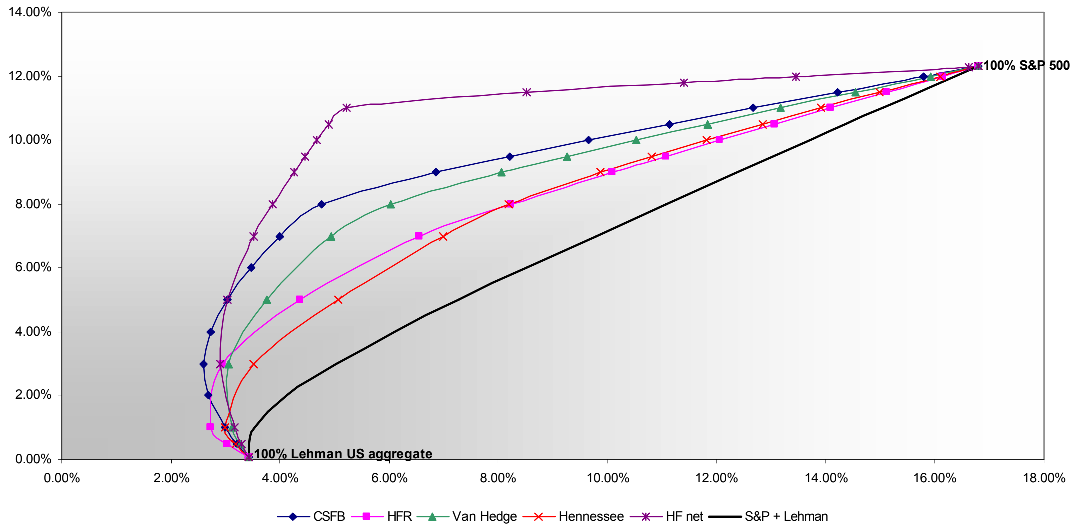
Maximal differences in the monthly returns of alternative indices (1998 – 2000)

Management Style	Maximal Difference in Monthly Returns
Convertible Arbitrage	4.75% (Oct 98; CSFB (-4.67) / Hennessee (0.08))
Emerging Markets	19.45% (Aug 98; (MARH -26.65) / Altvest (-7.2))
Equity Market Neutral	5.00% (Dec 99; Hennessee (0.2) / Van Hedge (5.2))
Event Driven	5.06% (Aug 98; CSFB (-11.77%) / Altvest (-6.71))
Fixed Income Arbitrage	10.98% (Oct 98; HF Net (-10.78) / Van Hedge (0.2))
Global Macro	17.80% (May 00: Van Hedge (-5.80) / HF Net (12))
Long/Short	22.04% (Feb 00: EACM (-1.56) / Zurich (20.48))
Merger Arbitrage	1.85% (Sep 98: Altvest (-0.11) / HFR (1.74))
Relative Value	10.47% (Sep 98: EACM (-6.07) / Van Hedge (4.40))
Short Selling	21.20% (Feb 00: Van Hedge (-24.3) / EACM (-3.09))
Distressed Securities	7.38% (Aug 98: HF Net (-12.08) / Van Hedge (-4.70))
Fund of Funds	8.01% (Dec 99: MAR-Zurich (2.41) / Altvest (10.42))
Global	18.29% (Dec 99: CSFB (0.09) / Magnum (18.38))

These differences in returns can, as we see, be greater than 20%! And it is obviously not the same thing for a "long/short" manager to be compared in February 2000 to a benchmark with a performance of –1.56% (EACM) or a benchmark with a performance of 20.48% (Zurich)! We should note that all the data series used in this article to calculate returns are given in US dollars.

The presence of a high level of heterogeneity in the different alternative indices has immediate strategic allocation consequences for an investor who wishes to use the alternative class to diversify a stock and/or bond portfolio. As an example, the following graph shows the efficient frontiers obtained through an optimal combination of the traditional S&P 500 (stock market) and Lehman Brother Global Bond (bond market) indices with different alternative fixed-income arbitrage indices on the basis of monthly data for the period from January 1996 to October 2001.

Efficient frontiers using stocks (S&P 500), bonds (LBGBI) and the Fixed Income Arbitrage Style (01/1996 – 10/2001)



The different problems posed by alternative indices, together with possible solutions, are discussed in detail in Schneeweis et al. (2001), Amenc and Martellini (2001) and Fung and Hsieh (2001).

II.3 Impact of bias in the database

The decision to post the performance of alternative funds in one of the competing databases (TASS, MAR, HFR) is purely voluntary and only a certain number of funds decide to participate. This leads to "**self reporting bias**" and since the funds that have refused to report to any of the databases are by definition unobservable, it is not possible to evaluate the impact of this bias. Certain funds choose not to publish their performance because the performance does not appear satisfactory, others because they have already reached their critical size. It is therefore difficult to know whether this bias has a positive or negative impact on the performances announced.

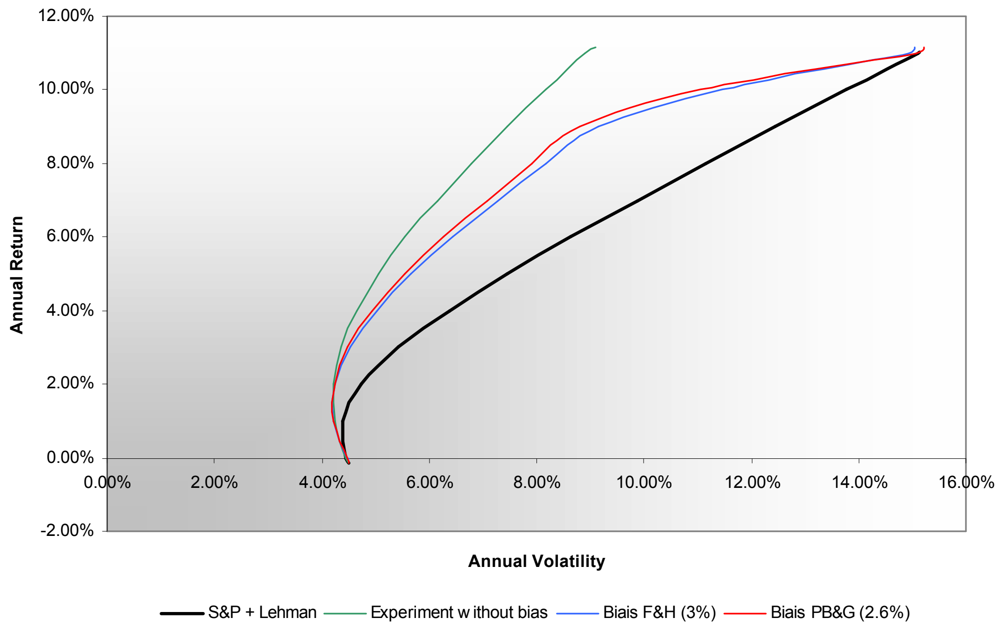
Since hedge funds that have performed poorly leave the industry, the funds that are still present in a database tend to be funds that have performed better than the average of the whole population. In this case we speak of "**survivorship bias**." Fung and Hsieh (2001) valued the average impact of this bias at 3.0%, compared to 2.6% for Park, Brown and Goetzmann (1999). The various databases are affected in different ways by this bias. For example, the TASS database has a higher survivorship bias than the HFR database because it has a default rate that is higher than HFR's.

The funds also have selection criteria that can be very different from one fund to the next, and the data provided will not be representative of the same management universe. This is referred to as "**selection bias**." For instance, HFR excludes managed futures from its databases while TASS and MAR take them into account. Most funds are present in one but not the other: of the 1,162 HFR funds and the 1,627 TASS funds, only 465 are common to both databases. 59% of the funds that are still in activity and 68% of the funds that no longer report to HFR are not part of the TASS database (cf. Liang (2001)). Fung and Hsieh (2001) valued the impact of this bias at 1.4 %, compared to 1.9 % for Park, Brown and Goetzmann (1999).

Out of the 465 funds in common between the HFR and TASS databases, only 154 (or 33.1%) have been included in both databases at the same time. However, when a fund is added to a database, all or part of its historical data is recorded ex-post in the database. Since it is in the funds' interest to display the most positive performance possible, it is probable that the mean performance displayed by the funds during their incubation period will be better than that of funds that have belonged to the corresponding database for a long time. In this case we talk about "**instant history bias**." Fung and Hsieh (2001) valued the impact of this bias at 1.4 % per year. If the funds are not recorded at the same date in two different databases, it is probable that the two databases will not be exposed to "instant history bias" in the same way. This risk is heightened by the fact that only 47% of the performances recorded are strictly identical.

To test the impact of the biases, and in particular the survivorship bias, in an optimal selection approach for portfolios that include the alternative class, we have generated efficient frontiers that integrate a survivorship bias estimated at 2.6%, which is consistent with the results of Fung and Hsieh (2001). Moreover, the graph below was generated under the assumption of an annual default rate for alternative funds of 8.3% (see Appendix 2 for justification and more details on this subject). The portfolios use the S&P 500 as a proxy for the stock class, the Lehman Global Bond Index as a proxy for the bond class, and the Global Tremont Index as a proxy for the alternative class.

Survivorship bias and efficient frontiers using stocks (S&P 500), bonds (LBGBI) and hedge funds (Global Tremont Index) (01/1996 – 05/2002)



These results reveal lesser benefits from alternative fund diversification than those obtained in the framework of a classic mean-variance analysis without integrating the survivorship bias. The efficient frontiers that integrate the alternative class continue nonetheless to dominate the efficient frontier obtained from the traditional classes (stocks and bonds). Furthermore, it is especially interesting to note that the alternative class allows the efficient frontier to be improved by reducing the global risk of the portfolio (i.e. "Risk Reducer"). This is in stark contrast to the alarmist discourse of those who continuously warn investors of the numerous risks of alternative investment.

II.4 Dynamic and non-linear dimensions of alternative risks

Most hedge fund managers follow dynamic investment strategies that distinguish them from the buy-and-hold type strategies often practised in traditional investment management. Moreover, the use of static positions in derivative and optional instruments reinforces the non-linear and dynamic character of the alternative strategies (see Fung and Hsieh (1997)). However, it is well known that risk measures such as the beta or the Sharpe ratio do not allow for adequate evaluation of dynamic and non-linear risks (see for example Dybvig (1988a, 1988b), Leland (1999) or Lo (2001)).

Nevertheless, in spite of this inappropriateness, the Sharpe ratio is still the most widely used measuring instrument for evaluating the risk-adjusted return of alternative investments. In a recent study carried out by Edhec (2002), it is revealed as the measure that is the most frequently used by distributors of hedge funds (notably funds of funds) to promote the superiority of alternative class returns.

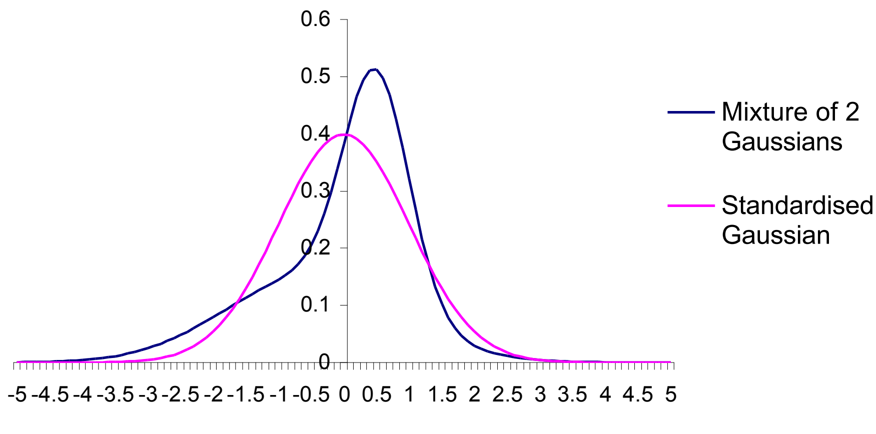
Indicators used in comparing the performance of European funds

Indicator	Total number of quotations (1)	Number of favoured quotations (2)
Sharpe Ratio	100%	93%
Sortino Ratio	28%	3%
M² and SRAP	19%	NS
Return/VaR	9%	2%
Others (including IR)	14%	2%

Source: Edhec (2002)
(1) several answers possible
(2) use of the indicator as a favoured risk-adjusted return measure in the sales literature.

This omnipresence of the Sharpe ratio poses a theoretical problem, to the extent that it assumes that investors are only interested in the first and second order moments of alternative fund return distributions, i.e. their mean and variance. This can only be justified, however, at the cost of simplistic assumptions relating either to the agents' utility function (quadratic utility, which does not, as we know, exhibit the desirable property of decreasing marginal utility), or to the return probability distribution (Gaussian (normal) distribution). However, the returns of alternative funds are clearly not Gaussian (see for example Brooks and Kat (2001)). In the case of portfolios that include derivative instruments, the assumption of Gaussian returns is not in fact tenable. Even if the return of the traditional asset class were Gaussian, the return of funds using derivative instruments or dynamic strategies relating to those traditional classes would not be. In point of fact, certain derivative instruments, such as options, generate final cash flows that are non-linear functions of the underlying assets, and it is well known that a non-linear function of a Gaussian variable is not distributed in a Gaussian manner.

As a result of taking their non-linear and non-Gaussian character into account, the investor generally displays a non-trivial preference for the third and fourth order moments of return distribution (skewness and kurtosis), as is evidenced, furthermore, by the development of measures of extreme risk such as the VaR (see below). It is possible to obtain two probability distributions with the same mean and the same variance, but with entirely different skewness and kurtosis. For example, an equally weighted mixture of a Gaussian with a mean of 0.5 and a standard deviation of 0.5 and a Gaussian with a mean of –0.5 and a standard deviation 1.32 exhibits the same mean and variance as a standardised Gaussian (0 and 1, respectively), but a skewness of –0.75 and a kurtosis of 6.06, compared to 0 and 3, respectively, for the standardised Gaussian (see graph below). Only by taking into account the third and fourth order moments can we truly distinguish the funds with returns that follow those probability distributions.



The use of the Sharpe ratio, besides its scientific character, which is open to criticism, also seems to us to be risk bearing. It may lead managers to implement "short volatility[9]" strategies based on the sale of "out of the money" put and/or call options. These strategies allow the volatility risk, measured by the second order moment of the return distributions, to be limited, while at the same time increasing its mean by cashing in premiums. By selling out of the money put options (strike price on average 7% below the market price) on the S&P 500, with a maturity of 3 months or less, Lo (2001) manages to obtain a Sharpe ratio of 1.94 for the period from January 1992 to December 1999 (compared to 0.98 for the S&P 500!). Of course, the downside of this strategy is the very significant increase in the risks of extreme loss, which only appear in moments greater than 2 (skewness and kurtosis) and which are therefore not taken into account in the Sharpe ratio. In Lo's example, the maximal loss recorded by the "Capital Decimation Partners" fund is 18.3%, compared to 8.9% for the S&P 500.

More generally, quite apart from these manipulations[10], it is possible to show, through a statistical model integrating fatter tails than those of the normal distribution, that minimising the second order moment (the volatility) is often accompanied by a significant increase in extreme risks and thus the Value-at-Risk (Sornette, Andersen, Simonetti, 2000).

There are two possible types of approach that allow the non-linearity of hedge fund returns to be taken into account within the framework of classic financial theory.

An initial approach, which requires a more significant methodological adjustment, involves using a non-linear model to explain the returns of alternative funds. For example, to model the non-linearity of returns, there has been a proposal to implement, in the decomposition of fund returns, alongside the normal and abnormal returns, a "phase-locking" type approach, representing both model risk in the case of extreme events and evolution in the correlations in the case of very significant movements in the markets (see Lo, 2001). From this perspective, the publication of conditional betas, by certain hedge fund promoters, already represents significant progress in taking the non-linear character of alternative returns into account.

Another approach, which is closer to the application framework of classic financial theory, involves using a linear model that employs non-linear regressors (or explanatory variables). Among the variables that allow the non-linearity of hedge fund returns to be represented, it seems natural to use portfolios of options.[11] Mitchell and Pulvino (2000) show that "merger arbitrage" strategies exhibit similar returns to those obtained through the sale of (naked) put options on the stock market index. At the same time, Fung and Hsieh (2000) note that "trend following" type strategies exhibit similar returns to those obtained through a combination of lookback type options. More generally, even though there is a potentially infinite number of different hedge fund strategies, Agarwal and Naik (2000) demonstrate that a dynamic portfolio of 3 options on the American Russell 3000 index with different strike prices enables a significant fraction of the returns of a fairly widespread class of hedge funds to be explained ex-post. Another possible choice of non-linear variables that allows hedge fund returns to be explained involves using hedge fund indices. This approach has significant practical advantages, even if it seems relatively unambitious from a conceptual point of view to attempt to explain the returns of hedge funds through the returns of other hedge funds. In this way, style analysis, inspired by the work of Sharpe (1992), and adapted for the alternative universe, allows the performance of an alternative fund to be analysed, in the absence of information on the strategy followed by the manager (see Lhabitant, 2001, and Amenc, Curtis and Martellini, 2002).

Finally, it should also be recalled that the Value-at-Risk, which is the most widely used composite indicator for extreme risk in the traditional universe, has also been subject to widespread criticism as to its appropriateness for analysing non-linear and dynamic strategies.

[9] cf. Anson (2000) and Lo (2001).

[10] Other more subtle manipulations of the Sharpe ratio can be envisaged. To this end, one could consult Spurgin (1999).

[11] We should note that the use of variables that allow implicit stock market volatility to be approximated, which we mentioned earlier, represents another way to take the optional and non-linear character of alternative fund returns into account.

A significant amount of literature[12] has commented widely on the limitations and necessary adaptations of the VaR for alternative investments:

- The VaR measures potential losses that occur normally or regularly; it has nothing to say about the consequences of exceptional events;
- Taking exceptional events into account exacerbates the statistical estimation problem. In the case of a VaR calculated from the distribution of past returns, it is necessary to have a very considerable amount of data to obtain a significant sample of "historical" VaR events. This problem, which already exists in the traditional universe, is exacerbated in the alternative universe by the frequency of the data, which is often monthly;
- The alternative approach, called "parametric," involves making an explicit assumption about the normality (or about a given law) of returns to calculate the VaR and, as such, is not appropriate for the alternative universe.

A solution to the criticism aimed at the historical and parametric VaRs has been proposed, with the VaR based on simulations that use the Monte Carlo method. This VaR has itself been subject to criticism, on the one hand because of the size of the simulations and therefore the size of the calculations required, and on the other because, frequently, it uses a normal distribution of the risk factor returns (semi-parametric VaR). The resulting simplification of the Monte Carlo simulations then contradicts the objective of overcoming the unrealistic initial framework of the parametric VaR.

Faced with these difficulties, investors and managers have implemented interesting solutions (stress testing, scenario analysis and more complex modelling of the distribution tails with extreme value theory).
These "variations" on the management and measurement of extreme risks should, in our opinion, be popularised and generalised and thereby permit a relative appreciation of the parametric risk and return measures. The latter are totally inappropriate for the alternative universe and this approach would allow for better management of the benefits of alternative diversification.

With that in mind, we present a pragmatic application of the VaR calculation in a fat tail distribution environment, along with its integration into an optimisation process (see Favre and Galinao (2000)). This method initially consists of calculating a VaR using a normal distribution formula and then a Cornish-Fisher expansion to take the skewness and kurtosis into account. Within the Gaussian framework, the VaR can be calculated explicitly by using the following formula:

$$P(dW \le -VaR) = 1 - a$$

$$VaR = n s W dt^{0.5}$$

where n = number of standard deviations at (1-α)
σ = annual standard deviation
W = current value of the portfolio
dt = fraction of the year

The analytical side of this normal VaR formula[13] was then adjusted using the Cornish-Fisher extension (1937) as follows:

$$z = z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2$$

where Zc = the critical value of the probability (1-α)
S = the skewness
K = the excess kurtosis (i.e. kurtosis minus 3)

[12] We could notably mention Chung (2000)
13 Mina and Ulmer, 1999, Delta-Gamma Four Ways and the RiskMetrics Group propose four VaR calculation methods for assets that are non-normally distributed: Johnson transformations, Cornish-Fisher expansions, Fourier methods and partial Monte-Carlo. They found that Cornish-Fisher is fast and easy to follow but sometimes lacks accuracy for extreme distributions.

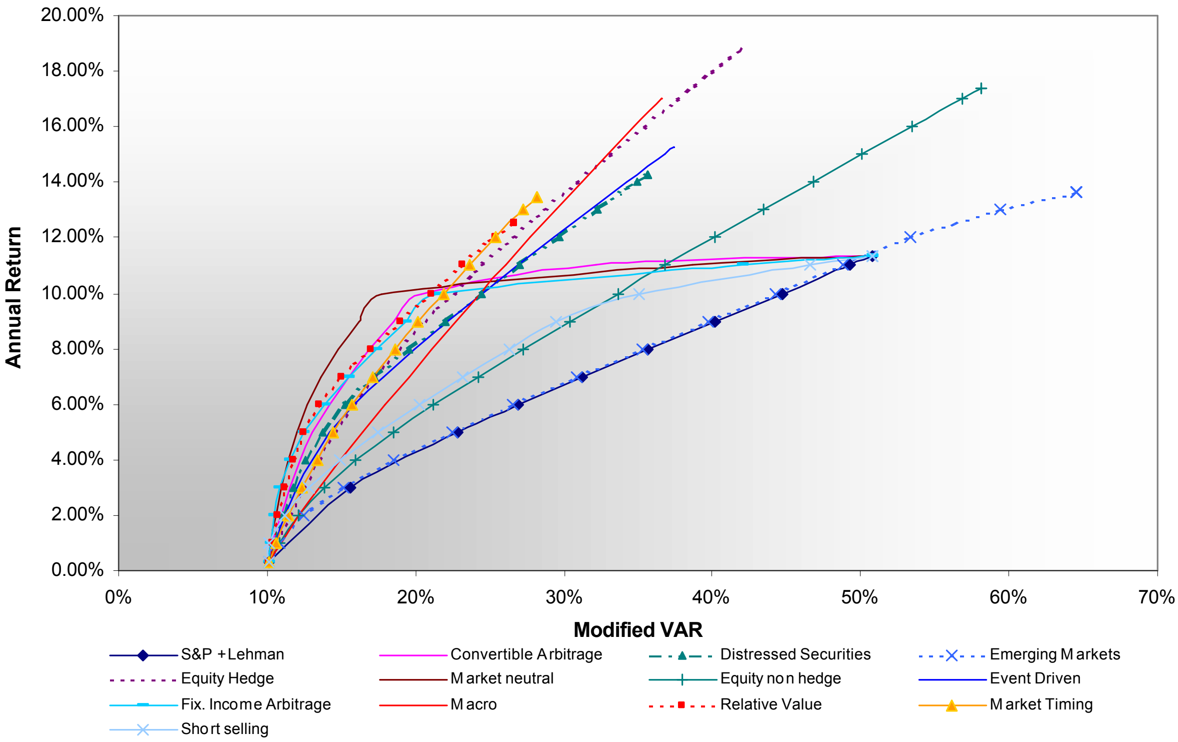
The adjusted VaR is therefore equal to:

$$VaR = W(m - zs)$$

It should be noted that if the distribution is normal, S and K (represents the excess kurtosis in the formula) are equal to zero and consequently, z=Zc, and we come back to the Gaussian VaR.

We carried out an efficient frontier calculation in a mean-VaR space, where we use the VaR at a threshold of 99% integrating the Cornish-Fisher correction, allowing investors' aversion to the extreme risks related to alternative investment to be taken into account. These efficient frontiers were calculated from HFR style performances over the period February 1990 - March 2002.[14]

Modified (Cornish-Fisher) mean/VaR efficient frontiers using stocks (S&P 500), bonds (LBGBI) and hedge fund styles (HFR) (02/1990 – 03/2002)

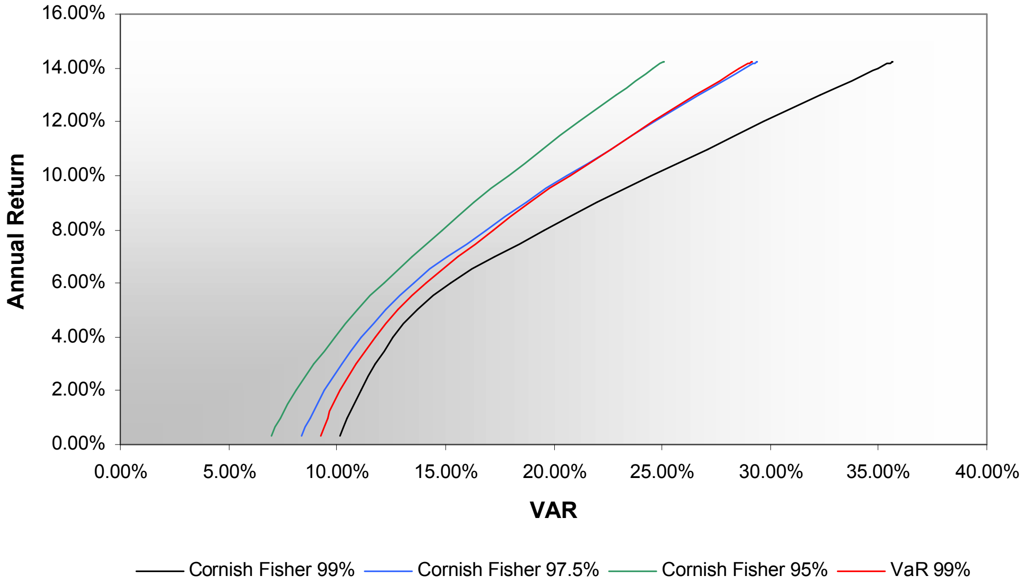


It is interesting to add that, according to our research, the efficient frontiers obtained using a Gaussian parametric VaR without a Cornish-Fisher correction for a 99% threshold are very close to those obtained with a VaR adjusted according to the Cornish-Fisher extension, but at a 97.5% threshold. We can therefore consider that investors who only take first and second order moments into account greatly underestimate (a factor of 2.5) the extreme risk to which they are exposed.

[14] We could also integrate the estimated survivorship bias for each alternative fund style (cf. part II.3).

As an example, we shall consider the case of "distressed securities" strategies.

Comparison of mean/VaR optimisations in the case of "distressed securities" type strategies (HFR) for the period (02/1990 – 03/2002)



What is more, it is appropriate to insist on the limitations of quantitative approaches in the area of risk monitoring and control. The risk linked to alternative funds should imperatively be evaluated within a framework of thorough analysis.

III. Conclusion: What attitude to adopt with regard to alternative investment risks?

Contrary to what certain hedge fund promoters would lead us to believe, alternative investment, even when it involves non-directional strategies with zero-beta, or very low volatility, presents risks. That is in fact its principal strength!

Good quality portfolio diversification is based on assets that are exposed to different risks. The alternative class, through the strategies and instruments it contains, provides this difference.

Obviously, modelling and measuring these new risks, and therefore controlling them, presents several challenges.

It would also appear to be prejudicial for the development of the hedge fund industry and the recognition of alternative investment as a fully-fledged asset class to neglect those challenges.

Through too great a desire to prove the superiority of hedge fund alphas and by forgetting certain sources of risk or the difficulty in measuring those risks, there is a danger of asset management professionals overselling an alternative class performance that is related to the prevailing economic situation. That would not stand up against the occurrence of events that would highlight the specific risks of those investments.

This would be all the more regrettable since, as we have shown, even when taking extreme risks, difficult market conditions and data bias into account, alternative investment retains, for all that, considerable advantages in the area of effective portfolio diversification.

References

- Agarwal, V., and Naik N., 2000a, Multi-period performance persistence analysis of hedge funds, *Journal of Financial and Quantitative Analysis*, 35, 327-342.
- Agarwal, V., and Naik N., 2000b, Performance evaluation of hedge funds with option-based and buy-and-hold strategies, *working paper*, London Business School.
- Amenc, N., Curtis S., and Martellini L., 2002, The alpha and omega of hedge fund performance measurement, *working paper*, Edhec Risk and Asset Management Research Center.
- Amenc, N., and Martellini L., 2001, The brave new world of hedge fund indexes, *working paper*, Edhec/Misys multi-style/multi-class research program.
- Anson, M. J. P., 2000, Hedge fund risk management for institutions, in *Managing hedge fund risk,* Risk Books.
- Asness, C., Krail, R. and Liew J., 2000, Do hedge funds hedge?, *working paper*, AQR Capital Management.
- Basle Committee on Banking Supervision, 1999, Credit risk modelling: current practices and applications, available on the BIS homepage http://www.bis.org
- Black, F., and Scholes M. S., 1973, The pricing of options and corporate liabilities*, Journal of Political Economy,* 81, 637-654.
- Black, F., 1972, Capital market equilibrium with limited borrowing, *Journal of Business*, 45, 444-454.
- Bohn, J. 1999a, Empirical assessment of a simple contingent-claims model for the valuation of risky debt, *working paper,* Haas School of Business.
- Bohn, J. 1999b, A survey of contingent claim approaches to risky debt valuation, *working paper,* Haas School of Business.
- Brooks, C., and Kat H., 2001, The statistical properties of hedge fund index returns and their implications for investors, *working paper*, The University of Reading, ISMA Centre.
- Capital Market Risk Advisors, Inc., July 9 2001, NAV/Fair value practices survey results.
- Chung, S. Y., 2000, What is the optimal portfolio risk measurement? A review of value-at-risk, in *Managing hedge fund risk,* Risk Books.
- Cornish, and Fisher, 1937, Moments and cumulants in the specification of distributions, Review of the International Statistical Institute, p. 307-320.
- Dybvig, P., 1988a, Inefficient dynamic portfolio strategies, or how to throw away a million dollars in the stock market, *Review of Financial Studies*, 1., 67-88.
- Dybvig, P., 1988b, Distributional analysis of portfolio choice, *Journal of Business*, 61, 369-393.
- Edhec, 2002, European Best practices in asset management, *preliminary findings*, Edhec Risk and Asset Management Research Center.
- Favre, L., and Galinao, J. A., 2000, Portfolio Allocation with Hedge Funds - Case Study of a Swiss institutional investor, working paper.
- Fung, W., and Hsieh D., 1997, Empirical characteristics of dynamic trading strategies, *Review of Financial Studies*, 10, 275-302.
- Fung, W., and Hsieh D., 2000, The risk in hedge fund strategies: theory and evidence from trend followers, *Review of Financial Studies*, forthcoming.
- Fung, W., and Hsieh D. A., 2001, Benchmark of hedge fund performance, information content and measurement biases, forthcoming, *Financial Analysts Journal*.
- Gordy M., 2000, A comparative anatomy of credit risk models, *Journal of Banking and Finance*, 24, 119-149.
- Grouhy, M., Galai, D., Mark, R., 2000, A comparative analysis of current credit risk models, *Journal of Banking and Finance*, 24, 59-117.
- Kao, D. L., July-Aug. 2000, Estimating and pricing credit risk: an overview*, Financial Analysts Journal*, Vol 56, n° 4, 50-66.
- Leland, H., 1999, Beyond mean-variance: risk and performance measures for portfolios with nonsymmetric distributions, *working paper*, Haas School of Business, U.C. Berkeley.
- Lhabitant, F., 2001, Hedge fund investing: a quantitative look inside the black box, *internal paper*, Union Bancaire Privee.
- Liang, B., 2001, Hedge fund performance: 1990-1999, *Financial Analysts Journal*, January/February, 11-18.

- Ljung, G.M and Box, G.E.P., 1978, On a measure of lack of fit in time series models, *Biometrika*, Vol. 65, n° 2, 299-315.
- Lintner J., 1971, The effect of short selling and margin requirements in perfect capital markets, *Journal of Financial and Quantitative Analysis*, 6, 5, 1173-1195.
- Lo, A., Nov-Dec. 2001, Risk management for hedge funds, introduction and overview, forthcoming, *Financial Analysts Journal*, 16-23.
- Longin F., Solnik B., 1995, Is the correlation in international equity returns constant?, *Journal of International Money and Finance*, 14.
- Markowitz, H., 1952, Portfolio selection, *Journal of Finance*, 7, 77-91.
- Merton, R.C., 1974, On the pricing of corporate debt: the risk structure of interest rates, Journal of Finance, 29, 449-470.
- Merton, R. C., 1973, An intertemporal capital asset pricing model, *Econometrica*, 41, 867-888.
- Mitchell, M., and Pulvino T., 2000, Characteristics of risk in risk arbitrage, working paper, Harvard Business School.
- Park J., S., Brown, and W. Goetzmann, 1999, Performance benchmarks and survivorship bias for hedge funds and commodity trading advisors, *Hedge Fund News*, August 1999.
- PWG, Hedge funds, leverage, and the lessons of Long-Term Capital Management, report of the President's Working Group on Financial Markets
- Ross, S., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory*, 343-362.
- Saunder, A., 1999, Credit risk measurement: new approaches to value at risk and other paradigms, *Wiley Frontier in Finance*, John Wiley & Sons.
- Schneeweis, T., Kazemi H. and Martin G., 2001, Understanding hedge fund performance, research paper, Lehman Brothers.
- Schneeweis, T., and Spurgin R., 1999, Multi-factor models in managed futures, hedge fund and mutual fund estimation, working paper, CISDM, University of Armhest.
- Sharpe, W., 1992, Asset allocation: management style and performance measurement", *Journal of Portfolio Management*, 18, 7-19.
- Sharpe, W., 1964, Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.
- Sornette D., J., Andersen, V. and Simonetti P., Portfolio Theory for "Fat Tails", International Journal of Theoretical and Applied Finance 3 (3), 523-535 (2000)
- Spurgin R., 1999, How to game your Sharpe ratio, Clark University, *working paper*.
- Watts, D., 1999, Small worlds: the dynamics of networks between order and randomness, Princeton University Press.
- Watts, D. and Strogatz, S., June 1998, Collective dynamics of 'small-world' network, Nature, vol. 393, n° 6, 440-442.

Appendix 1: Definition of hedge fund styles according to the HFR classification

Convertible arbitrage	Investment in convertible bonds. The strategy is to buy the convertible bond and sell short the common stock of the same company.
Distressed securities	Involves buying back, at a low price, the securities of companies that are experiencing financial difficulties. The securities targeted may cover a wide range, from senior secured debt (lowest risk) to common stock (highest risk).
Emerging markets	Investment in equities or bonds of emerging markets.
Equity hedge	Involves investing mainly in equities and derivative products. The manager systematically uses short selling, but takes care to maintain a permanent overall net position that is either long or neutral.
Market neutral	Exploits inefficiencies in the market through balanced buying of undervalued securities and selling of overvalued securities enabling either a beta or a dollar neutral approach to be obtained.
Equity non-hedge	Mainly involves holding long positions in equities, although managers may invest in derivative instruments and engage in short selling.
Event driven	Investment strategy that exploits price movements related to the anticipation of events affecting the life of the company (merger, acquisition, bankruptcy, etc.)
Fixed income arbitrage	The investment return is based on exploiting price anomalies related to interest rate instruments.
Macro	Investment strategy with a strong leverage effect on market events or developments.
Relative value	The objective of this type of strategy is to take advantage of the relative price differentials between related instruments.
Short selling	Maintains a net or simple short exposure relative to the market.
Market timing	Consists of investing systematically in assets for which an upward trend is anticipated.

Appendix 2: Efficient frontiers in the presence of survivorship bias

The object of this appendix is to describe the methodology used to generate efficient frontiers in the presence of bias in performance reporting. As we pointed out in section I, survivorship bias can have a considerable influence on the shape of the efficient frontier. Below, we explain the manner in which we have proceeded in order to take that into account.

We can firstly correct, as is classic in the literature, the estimation of mean returns to take account of survivorship bias. To do this, we initially estimate the historical mean return of the alternative class (sample mean). We then subtract a value that corresponds to the estimation of the survivorship bias carried out by Fung and Hsieh (2001), i.e. 3%, and Park, Brown et Goetzmann (1999), i.e. 2.6%. The variance-covariance matrix is also affected by the presence of survivorship bias. We must therefore, at a second stage, correct the volatility of the performances of the alternative class. We propose to take this into account through the following model, blending a Poisson process with the return law.

To simplify, we consider a problem with 3 assets: a stock index, a bond index and an alternative fund. The presence of survivorship bias leads us to replace the return distribution of the alternative fund i R_i with $1_{\{t_i > T\}} R_i$, where t_i represents the uncertain eventual disappearance date of the fund and $1_{\{A\}}$ represents the indicator function of event A. Therefore, if the fund does not disappear before the investment's time horizon T, the investor has a return that we will assume follows a classic normal distribution. However, if the fund disappears, the investment is suddenly reduced to 0. We complete the model by assuming that the disappearance date is distributed according to a Poisson process, independent from the R_i, with an intensity denoted as l_i. Thus, the probability of survival of the fund over the period [0,T] is approximately given by $e^{-l_i T}$. This probability can be estimated historically. As shown by Liang (2001), the choice of database used to calculate this probability has a notable influence on the results obtained. Therefore, even though the HFR database has an annual default rate of only 2.17%, the default rate of the TASS database is 8.3%. Since, for the moment, there is no consensus in the literature on the default rate to be selected, it seems coherent to take the study carried out by Liang as a reference. Nonetheless, Amin et Kat (2001) note that the default rate has tended to increase over the last few years. Consequently, we will correct the volatility of the alternative class by deliberately making two relatively pessimistic assumptions on the default rate for hedge funds: 8.3% (i.e. estimation carried out by Liang (2001) with the TASS database) and 19% (i.e. the default rate observed by Liang (2001) for CTA).

We also assume that the investor has a quadratic utility. Therefore, while this distribution for the alternative fund is not Gaussian (convolution of a Gaussian and a Poisson process), the optimal decision rule only takes the variance of the assets' return distribution into account as a risk measure. In short, it involves calculating the variance of the return distribution for alternative funds exposed to survivorship risk, together with its covariance with the traditional asset classes.

To do that, we use the variance decomposition formula:

$$Var\left(1_{\{t_i > T\}} R_i\right) = E\left[Var\left(1_{\{t_i > T\}} R_i \middle| t\right]\right] + Var\left[E\left(1_{\{t_i > T\}} R_i \middle| t\right]\right] = e^{-l_i T} Var(R_i) + \left[e^{-l_i T}\left(E(R_i)\right)^2 - e^{-2 l_i T}\left(E(R_i)\right)^2\right]$$

We finally obtain

$$Var\left(1_{\{t_i > T\}} R_i\right) = e^{-l_i T}\left[E\left((R_i)^2\right) - \left(E(R_i)\right)^2\right] + \left[e^{-l_i T}\left(E(R_i)\right)^2 - e^{-2 l_i T}\left(E(R_i)\right)^2\right]$$

or

$$Var\left(1_{\{t_i > T\}} R_i\right) = e^{-l_i T} E\left((R_i)^2\right) - e^{-2 l_i T}\left(E(R_i)\right)^2 \qquad (1)$$

It is also necessary to consider terms of the following type:

$$Cov\left(1_{\{t_i>T\}}R_i, R_j\right) = E\left[Cov\left(1_{\{t_i>T\}}R_i, R_j\right)\middle| t_i\right] + Cov\left[E\left(1_{\{t_i>T\}}R_i\right)\middle| t_i, E\left(R_j\right)\middle| t_i\right]$$

where R_j represents the return of a traditional stock or bond class.

We finally obtain (since the second term of the decomposition is null)

$$Cov\left(1_{\{t_i>T\}}R_i, R_j\right) = e^{-l_i T} Cov\left(R_i, R_j\right) \quad (2)$$

We then consider a portfolio invested in 3 asset classes, 2 traditional (stocks and bonds) and 1 alternative, in such a way that only the alternative fund is exposed to the survivorship risk. We further assume that the variance-covariance matrix associated with the 3 asset classes, obtained by integrating the modifications brought about by the survivorship risk (equations (1) and (2)), denoted Ω, is non-singular.

The efficient frontier associated with these 3 classes is defined as the location of the realisable portfolios with the smallest variance for a given expected return (or, in its dual formulation, as the set of realisable portfolios with the highest expected return for a given volatility).

The framework of the mean/Value-at-Risk analysis does not however allow for an explicit solution, to the extent that the Value-at-Risk of a portfolio is not written as a quadratic function of the portfolio weights. We therefore carry out a numerical resolution. However, the remarks relating to the estimation of the inputs, with the adjustment to take survivorship bias into account, remain valid.

The results that we obtain when we correct the return and the volatility of the alternative class for survivorship bias are presented in the graphs below. It is interesting to note that the correction we make to the volatility of the alternative class leads to a reduction in the risk of the portfolio. This correction also leads to a decrease in the covariance of the performances of the alternative class with the performances of the traditional indices. This is linked to the simplistic assumption that the Poisson process modelling the default is independent from the return distribution of the traditional assets. Therefore, adding a random variable that is independent from the returns of the asset prices leads to an increase in the decorrelation between the alternative class and the traditional class, and thus leads to a decrease in the total volatility of the portfolios. However, it is likely that the default process would not be independent from the returns on the stock and bond indices. We could imagine, for instance, that in countries where growth is low or negative, the risk of alternative funds defaulting is higher, at the very moment when stock markets are falling. In the absence of data on the dependence between the default risk and the return on traditional classes, it is nonetheless technically difficult to calibrate models taking that dependence into account. We therefore simply note that the model used tends to overestimate the diversification powers of the alternative class.

The consequence is therefore a slight improvement (shift towards the north-west) in the efficient frontier. This does not however compensate for the deterioration (i.e. shift towards the south-east) of the efficient frontier brought about by correcting the returns, regardless of the estimation chosen for the survivorship bias.