

Portfolio allocation with hedge funds: Case study of a Swiss institutional investor

Laurent Favre

Quantitative Investment Research, UBS

José-Antonio Galeano

Banque Cantonale Vaudoise

Abstract

Asset allocation advisers usually use the mean-variance framework to show the benefits of investing in hedge funds. The authors prove that this is not optimal when the assets are not normally distributed and develop a method based on a modified Value-at-Risk for non-normally distributed assets. We take the example of a Swiss pension fund investing part of its wealth in hedge funds and show that computing a portfolio with mean and variance considerably underestimates the risk of the portfolio.

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Introduction

The management of Swiss pension fund assets has undergone a radical change during the last decade as managers started to actively apply modern portfolio theory, which calls for international diversification across asset classes. As of 1985, the 'Loi sur la prévoyance professionnelle' (LPP) required that pension fund managers aim to achieve a minimum return of 4% and at the same time established regulations regarding asset categories and their maximum authorized weightings. Since the establishment of this law, the larger pension funds have moved towards more diversified portfolios with a further trend towards higher weightings in foreign equity and lower weightings in Swiss real estate. The year 2000 witnessed a major revision of the law, both in spirit and in substance, by giving managers more latitude in structuring their portfolios. However, more relevant for this study is that hedge funds are now considered a legitimate asset class that may be assessed through a diversified portfolio. This latter point will no doubt advance the process of diversifying into hedge funds and other alternative investments.

The aim of the study is to analyze the impact of hedge funds on a 'typical' Swiss pension fund portfolio satisfying the regulations established in the LPP. Our aim is to conduct a thorough and careful study that avoids replicating numerous other studies based on mean-variance optimization techniques and to take account of the special characteristics of hedge fund returns. In particular, it is well known in the industry that hedge fund returns are not normal, thereby violating one of the key assumptions underlying the use of simple mean-variance optimization.

Mean-variance framework

Mean-variance analysis has been the framework traditionally used for proving the benefits of international diversification. A number of recent studies have used this familiar framework to show that hedge funds can improve the risk/return profile on an internationally diversified portfolio. This mean-variance analysis developed by Markowitz, however, critically relies on two assumptions: either the investors have quadratic utility or the asset returns are jointly normally distributed. Both assumptions are not required, just one or the other.

Assumption 1: quadratic utility

If investors have quadratic preferences represented as below, mean-variance optimization is appropriate.

$$U(W) = aW - bW^2$$

If an investor has quadratic preferences, he cares only about the mean and variance of returns; and the skewness and kurtosis of returns have no effect on expected utility, i.e., he will not care, for example, about extreme losses. Quadratic utility has been shown to be inconsistent with observed human choice behavior with respect to risk.

Assumption 2: normal distribution

Mean-variance optimization can be justified if the returns of the assets are jointly normally distributed since the mean and the variance will completely describe the distribution. The normal distribution is symmetric (i.e. 50% of the probability lies above the mean), thus its skewness or third moment is 0. The kurtosis or fourth moment of a normal distribution has a value equal to 3.

If the distribution of hedge fund returns were jointly normal, and if we further had good estimates of the means, variances, and co-variances, then a mean-variance portfolio optimization would be appropriate. Unfortunately, as Table 1 below illustrates, (a) the sample distributions of hedge fund returns are skewed to both the left and the right and (b) the returns are leptokurtotic, meaning that the tails are fatter than the standard normal distribution.

Table 1 is based on data from January 1989 to June 1999 and includes the hedge funds index we constructed and used for our analysis as well as a sample of some of the funds included in the index. Please note that some data is not available for all the funds starting from January 1989, in which case we simply used data since inception of the fund.

The table clearly shows that the returns are not normally distributed thereby making the mean variance approach not an appropriate optimization technique.

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	Skewness	Kurtosis		Skewness	Kurtosis
Rocker Partners, L.P.	0.1	4.3	Omega	-3.0	14.0
Kodiak International Limited	0.7	5.7	Quantum	-0.2	3.1
Manhattan Investment Fund, Ltd.	0.9	5.8	Harch International Ltd.	-2.5	13.1
Symphony Allegro Fund	-0.1	3.4	Fairfield Sentry Ltd.	0.4	2.7
Centurion Capital International Ltd.	0.0	3.7	Springfield Synthetic Options Fund Ltd.	0.5	5.6
Ellington Mortgage Partners, L.P.	-5.7	40.9	WG trading Company L.P.	0.3	6.2
MKP Master Fund	-5.3	33.9	Hull Liquidity Fund, L.P.	-0.5	6.2
Trinity Fund	-2.3	14.2	SR Global Fund (Class A) Europe %	0.2	2.9
Lane Arbitrage Ltd. (INTL)	-1.1	7.8	Adelphi Europe Fund	-0.1	5.2
Watch Hill Fund L.P.	-1.6	9.3	Bayard Fund (Dollar)	-0.2	3.6
Stark Investments Limited Partnership	-2.8	13.1	Eureka U.S.	1.2	4.2
Kensington Global Strategies Fund Ltd.	0.4	4.6	Giano Capital USD Ltd.	0.2	3.2
Forest Fulcrum Fund L.P.	-2.4	9.5	Standard Pacific Capital Offshore Fund Ltd.	0.1	3.1
Highbride Capital Corporation	-0.3	9.9	Joho	0.6	4.6
Shepherd Investments International	-2.9	12.8	Maverick Fund Ltd.	1.0	9.8
Elliott Associates, L.P.	-1.2	6.4	Zweig-DiMenna Partners L.P.	-1.4	16.0
Comac Partners L.P.	-0.8	8.3	First Eagle Fund N.V. (Class A, B, and C)	-0.7	7.5
Cerberus Partners, L.P.	-0.5	5.6	FLA International Fund	0.3	3.4
Perry Partners, L.P.	-0.8	5.4	Pequot Partners Fund, L.P.	0.0	2.8
Canyon Value Realization Fund (CAYM)	-4.1	28.4	Standish Long/Short Limited Partnership	0.1	3.7
Deltec Emerging Market Equity L.P.	0.3	7.1	ACM Market Neutral Research Fund	-0.1	3.2
Coast Enhanced Income Fund II Ltd.	0.8	6.3	Kellner, Dileo & Co., L.P.	-2.7	15.0
Farallon Fixed Income Offshore Ltd. -A-	-4.9	32.7	Merger Fund Ltd.	-2.5	14.1
III Global Ltd.	-3.4	17.7	Levco Alternative Fund Ltd.	-0.4	4.0
Springfield Global Arbitrage Fund, Ltd.	0.1	6.4	West Broadway Partners, L.P.	-0.8	5.6
Jaguar	-0.1	4.9	York Investment Limited	-0.5	4.9
Moore	0.7	4.3	Our constructed HF portfolio	-1.4	7.7

Table 1

Source: Authors, Altvest.com

Modified value-at-risk approach

This framework substitutes the variance with the Value-at-Risk (VaR), which is another measure of risk. VaR corresponds to the amount of wealth that can be lost over a given period of time with a certain probability. Following Arzac and Bawa (1977) and others, we argue that decision-makers often make choices which are consistent with a mean-VaR approach, that is they prefer high mean returns but are highly averse to large losses. VaR has a number of advantages as a measure of risk:

- Downside risk measures may better reflect risk preferences of some investors.
- It has become an industry standard for measuring risk.
- It is easy to understand and to implement.
- It has been heavily researched.

- It is a single intuitive measure of risk.
- It can be easily extended to non-normally distributed returns.

Since hedge fund returns are skewed and fat-tailed we cannot use a VaR formula which assumes a normal distribution. One approach would be to use the sample distribution of returns for hedge funds as an estimate for the distribution of future returns. Another approach would be to assume a more appropriate probability distribution (e.g. the Student T distribution has fatter tails than the normal distribution, however it is not skewed). Instead we first calculate the VaR using the normal distribution formula and then use the Cornish-Fisher expansion to adjust the VaR for the estimated skewness and kurtosis. Recall that Value-at-Risk corresponds to the amount of wealth that can be lost over a given period of time with a certain probability:

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$$VaR = n\sigma W\Delta t^{Q^5}$$

$$Prob(dW \leq -VaR) = 1 - \alpha \quad (1)$$

where n = number of standard deviations at $(1-\alpha)^1$
 σ = annual standard deviation
 W = current value of the portfolio
 Δt = fraction of year

We adjust this normal VaR² formula for skewness and kurtosis analytically³, by using the Cornish-Fisher (1937)⁴ expansion as follows:

$$z_c = z + \frac{1}{6}(z^2 - 1)S + \frac{1}{24}(z^3 - 3z_c)K - \frac{1}{36}(2z^3 - 5z_c)S^2 \quad (2)$$

where Z = the critical value for probability $(1-\alpha)$, equals to -2.33 at 99%
 S = the skewness
 K = the excess kurtosis (= kurtosis minus 3)

The adjusted VaR is, therefore, equal to:

$$VaR = W(\mu - z_c\sigma) \quad (3)$$

Note that if the distribution is normal, S and K are equal to zero, thus $z = z_c$.

It is worth noting that the utility function underlying the mean-VaR approach is quite different from the quadratic utility function underlying the mean-variance approach. The latter is interested in higher return mean and lower return variance and smoothly trades off the two along the mean-variance efficient frontier. Let the 'safe' portfolio have a probability of falling below W^* equal to α and let the 'risky' portfolio have a probability of falling below W^* greater than α . A mean-VaR investor would prefer the safe portfolio to the risky portfolio, even if the risky portfolio mean return were much higher than the safe portfolio mean return!

Empirical results

■ Data

In order to calculate the mean-VaR efficient frontier, we first replicated a typical Swiss pension fund portfolio. For that purpose, we select the Swiss Performance Index (SPI) as a proxy for the Swiss equity asset class, the Lehman Brothers Swiss Bond Index for Swiss bonds, the Morgan Stanley Capital Index World (MSCI) for international equities, and the Lehman Brothers Weighted Global Bond Index for international bonds. We did not include investments in Swiss or foreign real estate (even though pension funds do invest in these assets) as sufficiently long return histories were not available. For the hedge fund asset class we use a self-constructed index intended to be replicable and representative of the hedge fund industry. We abstained from using the commercially available indices for a number of reasons to include unknown composition, general lack of transparency, and arithmetic averaging. Table 2 shows a summary of the different styles and number of managers used to construct our indices.

We constructed an initially equally weighted buy and hold Hedge Fund Global Index (HFGI) using monthly data from January 1989 to June 1999 on the 'largest, best known' open hedge funds. We would like to emphasize here that we were not constructing the optimal hedge fund portfolio for a pen-

Investment style	Number of hedge funds selected
Short only & short biased	3
Statistical arbitrage	2
Asset & mortgage	5
Convertible bond	5
Distressed securities	5
Emerging markets	1
Fixed income arbitrage	4
Global macro	4
High yield bond	1
Index & options arbitrage	4
Long-short European equity	5
Long-short International equity	2
Long-short U.S. equity	5
Market neutral	2
Merger arbitrage	5
Total	53

Table 2

1 In order to transform a monthly return standard deviation to an annual return standard deviation, the monthly standard deviation should be multiplied by the square root of 12.

2 Mina and Ulmer, 1999, Delta-Gamma Four Ways, Riskmetrics Group. They provide four methods to compute the VaR for non-normally distributed assets: Johnson transformations, Cornish-Fisher expansion, Fourier method, and partial Monte-Carlo. They found that Cornish-Fisher is fast and tractable, but sometimes not accurate with extremely sharp distributions.

3 David X Li, 1999, Value-at-Risk based on volatility, skewness, and kurtosis; Riskmetrics group, Working paper. He also derives an analytical formula for the confidence interval by using estimating functions. But, with his formula it was not possible to find a realistic one-side confidence level for negative returns.

4 Cornish and Fisher, 1937, Moments and cumulants in the specification of distributions, *Review of the International Statistical Institute*, p. 307-320.

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sion fund and as such did not select the funds based on their historical returns or correlations with broad indices.

The same analysis was repeated with the well-known Hedge Fund Research weighted composite index to test the robustness of the results to the choice of the hedge fund index. As the results are similar, we show only those results using our constructed HFGI.

Investment constraints

In order to replicate a typical pension fund portfolio structure, we incorporated the constraints from the legal restrictions in the LPP law. The legal constraints imposed on pension funds (OPP2, art.53) mainly affect 7 asset classes by defining maximum limits for each and, on a broader scale, their combinations. In our optimization, we therefore introduced the following investment limits:

Asset class	Limits
SPI	≤ 30%
MSCI	≤ 25%
SWGBI	≤ 20%
SSBI	≤ 100%
HFGI	≤ 10%

Based on discussions with Swiss pension funds and portfolio managers, and considering the legal restrictions in the LPP law, we assumed that the pension fund's portfolio weight in hedge funds cannot exceed 10%. Further, the optimization imposes full investment and prohibits shorting.

Results

In the first stage we computed the efficient frontier in a mean-VaR setting. To convert USD returns into CHF, we assumed that perfect hedging was feasible and at no cost⁵. We computed VaRs at the 99% confidence level using three techniques:

- We assumed that index returns (including the HFGI) are normally distributed and computed a normal VaR.
- We used the Cornish-Fisher expansion to calculate the adjusted VaR.

- We used the empirical distribution to compute the empirical VaR.

These three VaR techniques generate different mean-VaR efficient frontiers, as shown in Figure 1.

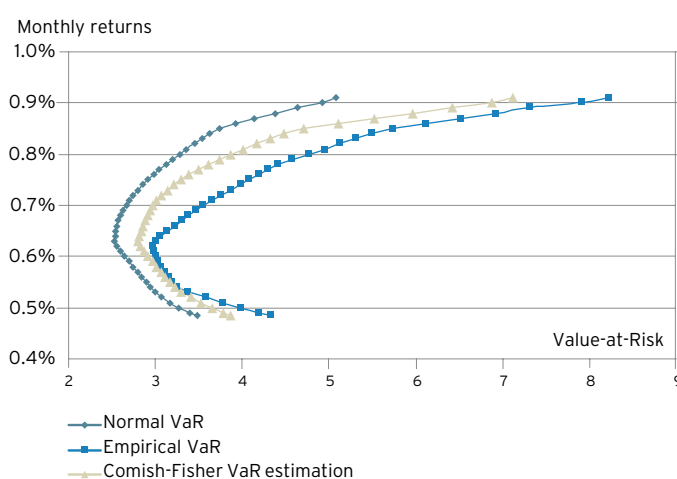


Figure 1: Normal, empirical, Cornish VaR estimation at 99%.
Source: Author, Altvest.com, Datastream. The HFGI portfolio weight is constrained to be less than 10%.

We see the effect of skewness and kurtosis of the hedge fund returns and also the other indices. The adjusted VaR and empirical VaR efficient frontiers are shifted to the right of the normal VaR efficient frontier. This suggests that if an investor ignores the skewness and kurtosis in asset class return distributions he may be overly optimistic about the mean return and/or of the VaR of his optimal portfolio. Since the adjusted VaR and empirical VaR use return distributions with longer left tails it is not at all surprising that their efficient frontiers indicate higher levels of risk at each level of mean returns.

We now provide evidence on whether the addition of hedge fund investments adds value to a typical pension fund portfolio, assuming that the pension fund risk is correctly measured by our VaR-based measures. We compare the efficient frontiers assuming maximum portfolio weights of 0.0%, 2.5%, 5%, 7.5% and 10% for hedge funds. For each constraint an effi-

⁵ i.e. the forward rate is an unbiased estimator of the future spot rate.

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cient frontier is computed. When the portfolio manager invests in hedge funds, the adjusted VaR at a 95% confidence level⁶ is computed. In all cases, the pension fund is not allowed to invest more than a total of 55% in the SPI, MSCI, and HFGI⁷.

The above figure compares the mean-modified VaR efficient frontiers for various maximum weights in the hedge funds portfolio. The results as illustrated in Figure 2 are quite similar to those results obtained using a mean-variance framework: adding hedge funds to a typical pension fund structure

improves its risk/return profile and provides valuable diversification effects when one takes into account skewness and kurtosis. Our results use a modified VaR as a risk measure and thus may provide assurance to a pension fund manager who is concerned that a hedge fund portfolio will increase his downside exposure. The evidence here suggests that a hedge funds portfolio can be used to reduce downside exposure even if the extreme returns are taken into account.

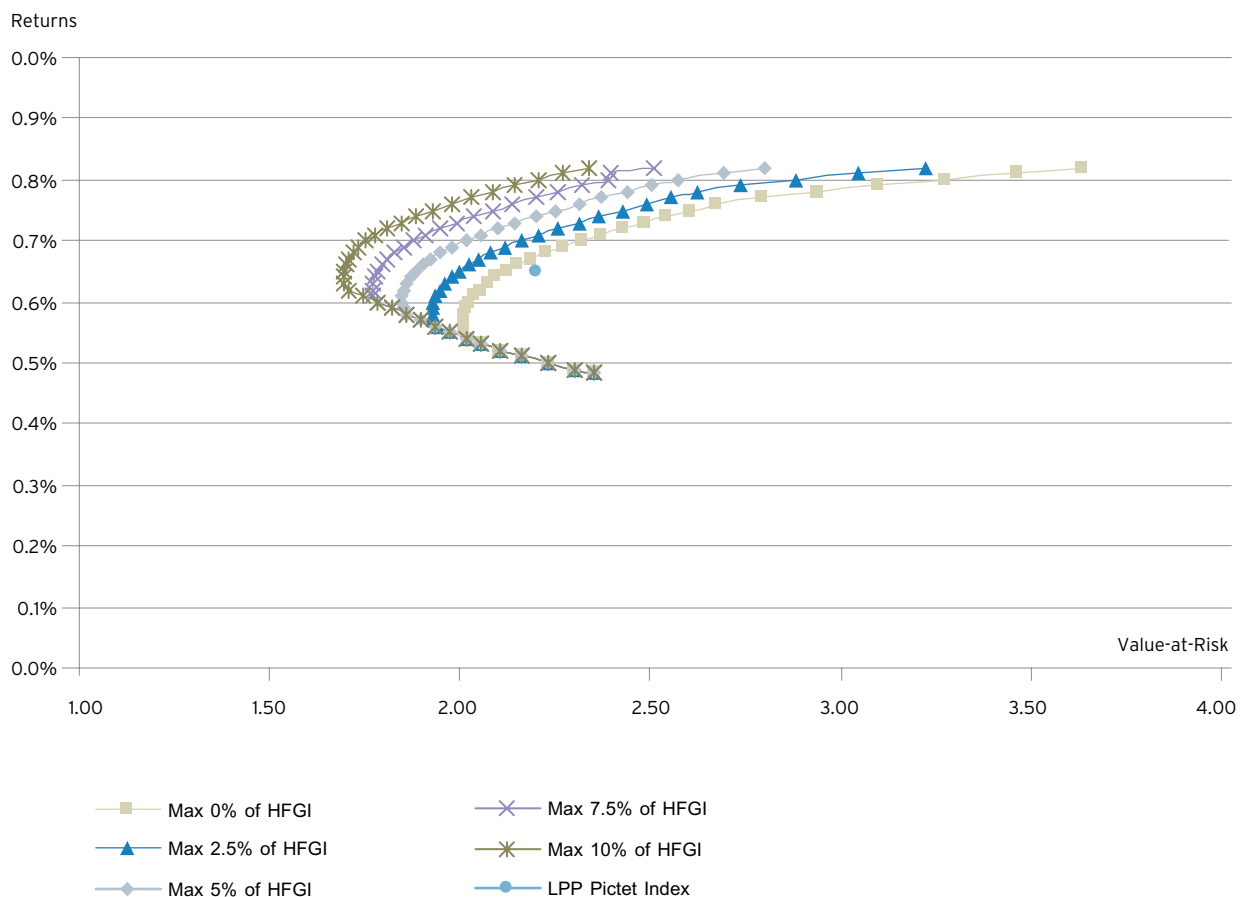


Figure 2: Efficient frontiers with and without HFGI

Source: Author, Altvest.com, Datastream. The HFGI portfolio weight is constrained to be less than 10%. The other four indices are bounded according to the Swiss law. Moreover, $W_{SPI} + W_{MSCI} + W_{HFGI} \leq 55\%$. The frontier takes into account the non-normality of the assets.

6 The graphs for the 97.5% and 99% confidence levels are similar.

7 In 1999, the Swiss law allows pension funds to invest a maximum of 30% in the SPI and 25% in the MSCI. The sum of both is 55%.

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Conclusion

Our approach to analyzing the effects of diversifying a Swiss pension fund portfolio with hedge funds (1) takes into account the underlying assumptions of the analysis framework, (2) models risk in a manner which we believe is consistent with the behavior of pension fund managers, and (3) incorporates constraints until now faced by managers. Assuming that the modified VaR is the appropriate measure of risk for a Swiss pension fund, the results showed that there are benefits to be obtained from investing in the hedge fund asset class. We assume that the portfolio of hedge fund is well diversified, especially during extreme negative market downturns.

Nevertheless, as hedge funds have other special features (a redemption delay, a higher attrition rate than stocks, high leverage, opacity, and rising correlations during market turmoil), a further analysis of these benefits such as stress testing (simulation) and local correlation analysis is clearly warranted. Rising correlations during market turmoil is of particular concern and is due to the concave payoff structure of many hedge fund strategies, except for short sellers, CTAs, and some arbitrage funds. Intuitively, by concave returns, we mean that returns are high when the market has small or medium absolute value returns, but low when the market has high absolute value returns. These observations imply that the strategies and the hedge funds in the hedge fund portfolio should be carefully selected to dampen the volatility of the rest of the portfolio.

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